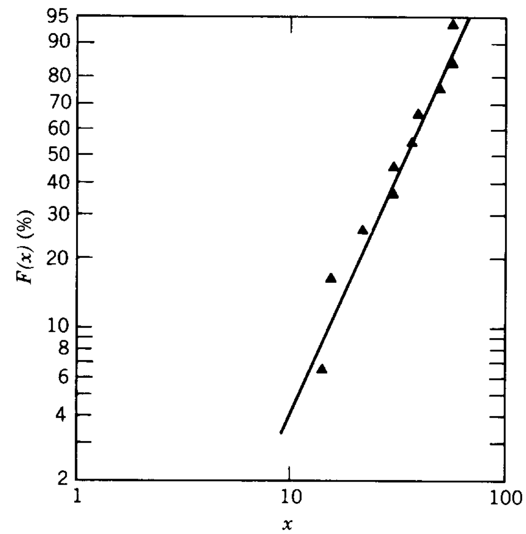
Example 11.1 Fatigue Life and Reliability of a Deep-Groove Ball Bearing

Problem Statement



A 209 DGBB yields a fatigue life of 100 million revolutions with 90% reliability. What fatigue life would be consistent with 95% reliability?

Problem Solution

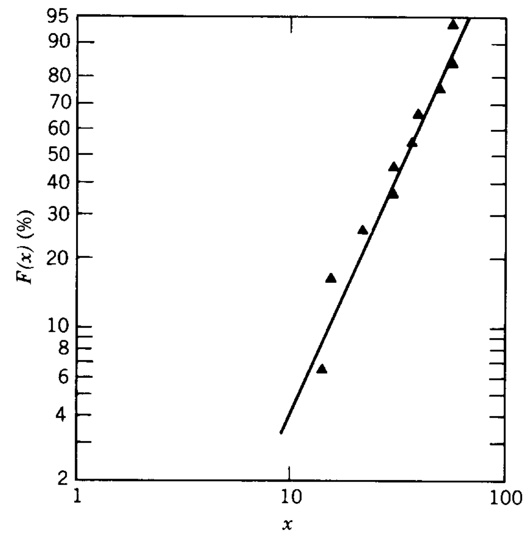
Eq.(11.21)

$$\ln\frac{1}{S}=\ln\frac{1}{0.95}=0.1053\left(\frac{L_S}{L_{10}}\right)^e=0.1053\left(\frac{L_5}{10^8}\right)^{10/9}$$

$L_5 = 52.2 \cdot 10^6$ **revolutions**

Example 11.2 Remaining Fatigue Life of a Group of Deep-Groove Ball Bearing

Problem Statement



30 of a group of 100 DGBBs have failed in fatigue.
Estimate the L_{10} life for the remaining 70 bearings.

Problem Solution

$$S_a = \frac{70}{100} = 0.70$$

Eq.(11.21)

$$\ln\frac{1}{S_a} = \ln\frac{1}{0.70} = 0.1053\left(\frac{L_a}{L_{10}}\right)^e = 0.1053\left(\frac{L_a}{L_{10}}\right)^{10/9}$$

$$L_a = 3.00L_{10}$$

After additional life of the 70 surviving brgs has been attained, the number of surviving bearings = 0.9 · 70 = 63. The relative number of surviving bearings = 63)100 = 0.63.

Eq.(11.21) $$\ln\frac{1}{S_b} = \ln\frac{1}{0.63} = 0.1053\left(\frac{L_b}{L_{10}}\right)^e = 0.1053\left(\frac{L_b}{L_{10}}\right)^{10/9}$$

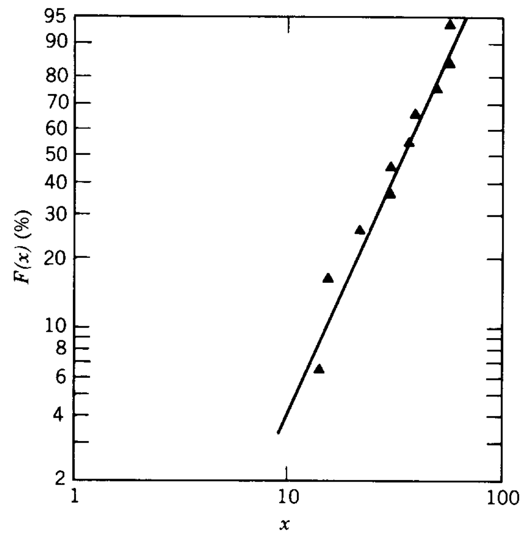
$$L_b = 3.79L_{10}$$

$$L_{10}' = L_b - L_a = 3.79L_{10} - 3.00L_{10} = 0.79L_{10}$$

Example 11.3 Additional Fatigue Life of a Group of Deep-Groove Ball Bearing after Some Have Failed.

Problem Statement

A group of DGBBs has an L_{10} life of 5000 hr. The bearings have been operated for 10000 hr and some have failed. Estimate the additional L_{10} life that can be expected from the remaining bearings.



Problem Solution

The relative number of bearings attaining or exceeding life L_a is S_a.

Eq.(11.21)

$$\ln\frac{1}{S_a} = 0.1053\left(\frac{L_a}{L_{10}}\right)^e$$

After the additional L_{10} life is attained, the relative number of bearings remaining is $S_b = 0.9\, S_a$ corresponding to life L_b.

Eq.(11.21)

$$\ln\frac{1}{S_b} = 0.1053\left(\frac{L_b}{L_{10}}\right)^e$$

Since $S_b = 0.9\, S_a$

$$\ln\frac{1}{S_b} = \ln\frac{1}{0.9} + \ln\frac{1}{S_a}$$

$$\ln\frac{1}{S_a} + 0.1053 = \left(\frac{L_b}{L_{10}}\right)^{e\cdot} \cdot 0.1053$$

By subtraction

$$0.1053 = 0.1053\frac{\left(L_b^e - L_a^e\right)}{L_{10}^e}$$

and

$$L_b = \left(L_{10}^e + L_a^e\right)^{1/e}$$

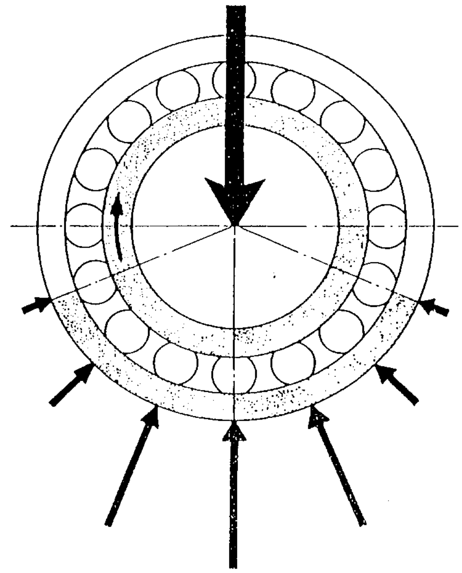
Additional life is given by $L_{10}^{'} = L_b - L_a$

$$L_{10}^{'} = \left(L_{10}^{e} - L_{a}^{e}\right)^{1/e} - L_a$$

$$L_{10}^{'} = \left\{(5000)^{10/9} + (10000)^{10/9}\right\}^{0.9} - 10000 = 4100hr$$

Example 11.4 Deep-Groove Ball Bearing Fatigue Life

Problem Statement



The 209 DGBB of Ex. 7.1 operates at shaft speed of 1800 rpm. Estimate the L_{10} life of the bearing.

Problem Solution

Eq.(11.67)

$$Q_{ci} = 93.2\left(\frac{2f_i}{2f_i - 1}\right)^{0.41} \frac{(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}} \gamma^{0.3} D^{1.8} Z^{-1/3}$$

$$Q_{ci} = 93.2\left(\frac{2\cdot 0.52}{2\cdot 0.52 - 1}\right)^{0.41} \frac{(1-0.1954)^{1.39}}{(1+0.1954)^{1/3}} (0.1954)^{0.3} (12.7)^{1.8} (9)^{-1/3} = 7058N$$

Eq.(11.67) $$Q_{co} = 93.2\left(\frac{2f_o}{2f_o - 1}\right)^{0.41} \frac{(1+\gamma)^{1.39}}{(1-\gamma)^{1/3}} \gamma^{0.3} D^{1.8} Z^{-1/3}$$

$$Q_{co} = 93.2\left(\frac{2\cdot 0.52}{2\cdot 0.52 - 1}\right)^{0.41} \frac{(1+0.1954)^{1.39}}{(1-0.1954)^{1/3}} (0.1954)^{0.3} (12.7)^{1.8} (9)^{-1/3} = 13970N$$

Ex.(7.1)

ψ	Q_ψ(N)
0	4536
±40°	2846
±80°	61
±120°	0
±160°	0

Eq.(11.68) $$Q_{ei} = \left(\frac{1}{Z} \sum_{j=1}^{j=Z} Q_j^3 \right)^{1/3}$$

$$Q_{ei} = \left\{ \frac{1}{9} \left[(4536)^3 + 2 \cdot (2846)^3 + (61)^3 + 2 \cdot 0 + 2 \cdot 0 \right] \right\}^{1/3} = 2475N$$

Eq.(11.70) $$L_i = \left(\frac{Q_{ci}}{Q_{ei}} \right)^3 = \left(\frac{7058}{2475} \right)^3 = 23.2 \cdot 10^6 \, revolutions$$

Eq.(11.74) $$Q_{eo} = \left(\frac{1}{Z} \sum_{j=1}^{j=Z} Q_j^{10/3} \right)^{0.3}$$

$$Q_{eo} = \left\{ \frac{1}{9} \left[(4536)^{10/3} + 2 \cdot (2846)^{10/3} + (61)^{10/3} + 2 \cdot 0 + 2 \cdot 0 \right] \right\}^{0.3} = 2605N$$

Eq.(11.75) $$L_o = \left(\frac{Q_{co}}{Q_{eo}} \right)^3 = \left(\frac{13970}{2605} \right)^3 = 154.4 \cdot 10^6 \, revolutions$$

Eq.(11.79)

$$L = \left(L_i^{-10/9} + L_o^{-10/9} \right)^{-0.9} = \left((23.2)^{-1.11} + (154.4)^{-1.11} \right)^{-0.9} \cdot 10^6 = 20.9 \cdot 10^6 \, revolutions$$

$$L = \frac{20.9 \cdot 10^6 \, revolutions}{1800 revolutions / \min \cdot 60 \min / hr} = 194 hr$$

Example 11.5 Angular-Contact Ball Bearing Fatigue Life

Problem Statement

The 218 ACBB of Ex. 7.5 is operated at 3600 rpm under a 17,800 N (4000 lb) thrust load. Estimate the L_{10} fatigue life of the bearing for inner ring rotation.

Problem Solution

Ex.(2.3) $D = 22.23$ mm (0.875 in.)

Ex.(2.6) $d_m = 125.3$mm (4.932 in.)

Ex.(7.4) $\alpha = 41.6°$

Ex.(7.5) $Z = 16$ balls
$Q = 1676$ N (376.6 lb)

Eq.(2.27)
$$\gamma = \frac{D\cos\alpha}{d_m} = \frac{22.23 \cdot \cos 41.6°}{125.3} = 0.1327$$

Eq.(11.67)
$$Q_{ci} = 93.2 * \left(\frac{2f_i}{2f_i - 1}\right)^{0.41} \frac{(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}} \left(\frac{\gamma}{\cos\alpha}\right)^{0.3} D^{1.8} Z^{-1/3}$$

$$Q_{ci} = 93.2 * \left(\frac{2 \cdot 0.5232}{2 \cdot 0.5232 - 1}\right)^{0.41} \frac{(1-0.1327)^{1.39}}{(1+0.1327)^{1/3}} \left(\frac{0.1327}{\cos 41.6°}\right)^{0.3} (22.23)^{1.8} (16)^{-1/3}$$

Q_{ci} = 16520 N (3713 lb)

***Angular-contact ball bearings with ∀E # 40E are classified as radial bearings and are rated using $f_m = 93.2$**

To account for spinning caused by the non-zero contact angle, the inner raceway capacity is reduced by the factor (1-0.33sinα).

$$Q_{ci}' = Q_{ci}(1 - 0.33\sin\alpha) = 16520(1 - 0.33\sin 41.6°) = 12{,}900\,N\,(2{,}899\,lb)$$

Eq.(11.70) $$L_i = \left(\frac{Q_{ci}}{Q_{ei}}\right)^3 = \left(\frac{12,900}{1676}\right)^3 = 456 \cdot 10^6 \, revolutions$$

Eq.(11.67) $$Q_{co} = 93.2 * \left(\frac{2f_o}{2f_o - 1}\right)^{0.41} \frac{(1+\gamma)^{1.39}}{(1-\gamma)^{1/3}} \left(\frac{\gamma}{\cos\alpha}\right)^{0.3} D^{1.8} Z^{-1/3}$$

$$Q_{co} = 93.2 * \left(\frac{2 \cdot 0.5232}{2 \cdot 0.5232 - 1}\right)^{0.41} \frac{(1+0.1327)^{1.39}}{(1-0.1327)^{1/3}} \left(\frac{0.1327}{\cos 41.6°}\right)^{0.3} (22.23)^{1.8} (16)^{-1/3}$$

Q_{co} = 26,180 N (5883 lb)

$$Q_{co}^{'} = Q_{co}(1 - 0.33\sin\alpha) = 26180(1 - 0.33\sin 41.6°) = 20,440 N (4,594 lb)$$

Eq.(11.75) $$L_o = \left(\frac{Q_{co}}{Q_{eo}}\right)^3 = \left(\frac{20,440}{1676}\right)^3 = 1814 \cdot 10^6 \, revolutions$$

Eq.(11.79)

$$L = \left(L_i^{-10/9} + L_o^{=10/9}\right)^{-9/10} = \left((456)^{-10/9} + (1814)^{=10/9}\right)^{-9/10} = 382.3 \cdot 10^6 \, revolutions$$

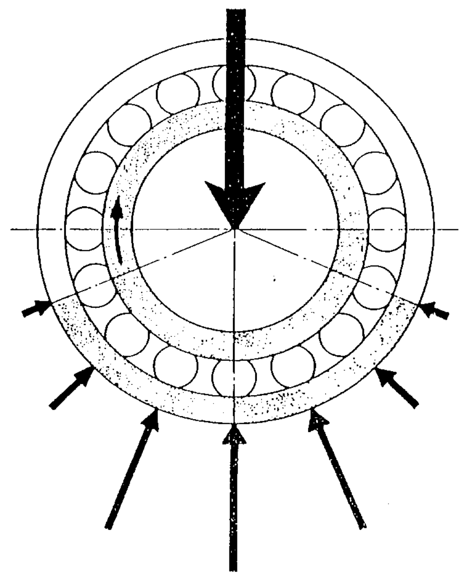
$$L = \frac{382.3 \cdot 10^6}{3600 \cdot 60} = 1,770 hr$$

Example 11.6 Fatigue Life of a Radial, Cylindrical Roller Bearing

Problem Statement

Assuming modified line contact and inner raceway rotation, estimate the L_{10} fatigue life of the 209 CRB of Ex. 7.3.

Problem Solution



Ex.(2.7)

D = 10 mm (0.3937 in.)
d_m = 65 mm (2,559 in.)
l = 9.6 mm (0.378 in.)
Z = 14 rollers

Ex.(7.3)

ψ	Q_ψ(N)
0°	1926
±25.71°	1355
…..	0
180°	0

Eq.(2.27) $$\gamma = \frac{D\cos\alpha}{d_m} = \frac{10\cos 0°}{65} = 0.1538$$

Eq.(11.143) $$Q_{ci} = 552\lambda_i \frac{(1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \gamma^{2/9} D^{29/27} l^{7/9} Z^{-1/4}$$

Table 11.4 **use $\lambda_I = \lambda_o = 0.61$**

$$Q_{ci} = 552 \cdot 0.61 \frac{(1-0.1538)^{29/27}}{(1+0.1538)^{1/4}} (0.1538)^{2/9} (10)^{29/27} (9.6)^{7/9} (14)^{-1/4} = 6381N$$

Eq.(11.144) $$Q_{ei} = \left(\frac{1}{Z} \sum_{j=1}^{j=Z} Q_j^4 \right)^{1/4}$$

$$Q_{ei} = \left\{ \frac{1}{14} \left[(1926)^4 + 2 \cdot (1355)^4 + ...0 \right] \right\}^{1/4} = 1100N$$

Eq.(11.145) $$L_i = \left(\frac{Q_{ci}}{Q_{ei}} \right)^4 = \left(\frac{6381}{1100} \right)^4 = 1132 \cdot 10^6 \, revolutions$$

Eq.(11.143) $$Q_{co} = 552 \lambda_o \frac{(1+\gamma)^{29/27}}{(1-\gamma)^{1/4}} \gamma^{2/9} D^{29/27} l^{7/9} Z^{-1/4}$$

$$Q_{co} = 552 \cdot 0.61 \frac{(1+0.1538)^{29/27}}{(1-0.1538)^{1/4}} (0.1538)^{2/9} (10)^{29/27} (9.6)^{7/9} (14)^{-1/4} = 9621N$$

Eq.(11.146) $$Q_{ei} = \left(\frac{1}{Z} \sum_{j=1}^{j=Z} Q_j^{9/2} \right)^{2/9}$$

$$Q_{eo} = \left\{ \frac{1}{14} \left[(1926)^{9/2} + 2 \cdot (1355)^{9/2} + ...0 \right] \right\}^{2/9} = 1148N$$

Eq.(11.147) $$L_o = \left(\frac{Q_{co}}{Q_{eo}} \right)^4 = \left(\frac{9621}{1148} \right)^4 = 4937 \cdot 10^6 \, revolutions$$

Eq.(11.148)

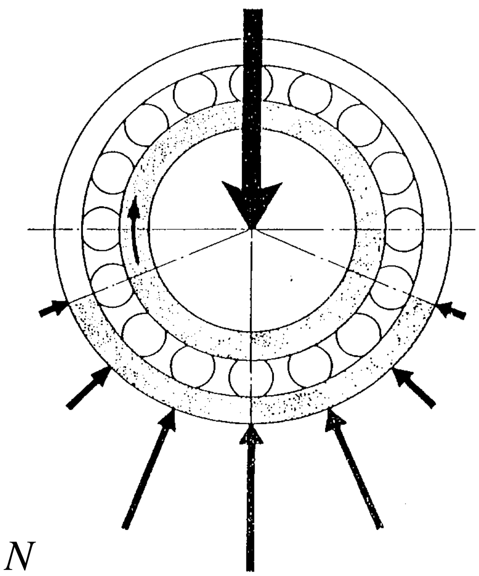
$$L = \left(L_i^{-9/8} + L_o^{-9/8} \right)^{-0.889} = \left((1155)^{-1.125} + (4937)^{-1.125} \right)^{-0.889} \cdot 10^6 = 985 \cdot 10^6 \, revolutions$$

Example 11.7 Standard Fatigue Life of a Radial, Deep-Groove Ball Bearing

Problem Statement

The 209 DGBB of Ex. 7.1 operates at shaft speed of 1800 rpm. Estimate the L_{10} life of the bearing using the Standard method.

Problem Solution



Ex.(2.1) $D = 12.7$ mm (0.50 in.)
$Z = 9$ balls

Ex.(2.5) $\gamma = 0.1954$

Table 11.12 at $\gamma = 0.1954$, $f_{cm} = 77.93$

Eq.(11.173) $C = f_{cm}(i\cos\alpha)^{0.7} Z^{2/3} D^{1.8}$

$$C = 77.93(1\cdot\cos 0°)^{0.7}(9)^{2/3}(12.7)^{1.8} = 32710N$$

$$F_e = F_r = 8900N$$

Eq.(11.108) $L = \left(\frac{C}{F_e}\right)^3 = \left(\frac{32710}{8900}\right)^3 = 49.64\cdot 10^6\, revolutions$

$$L = \frac{49.64\cdot 10^6\, revolutions}{1800 revolutions/\min\cdot 60\min/hr} = 459.6hr$$

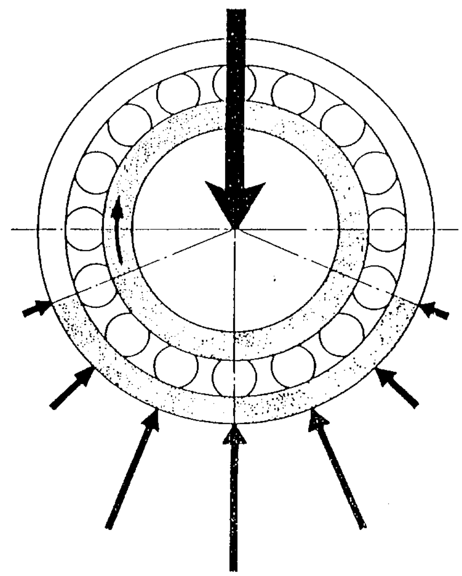
To compare this result with that of Ex. 11.4, it is necessary to multiply the result by b_m^3. From Table 11.11, $b_m = 1.3$. Accordingly, the comparable L_{10} life of Ex. 11.4 is 2.197 × 194 = 426.2 hr. It is evident that the Standard-estimated life is not as accurate as that calculated in Ex. 18.4. In this particular case, however, the difference is not significant.

Example 11.8 Standard Fatigue Life of a Radial, Cylindrical Roller Bearing

Problem Statement

The 209 CRB of Ex. 7.3 supports a radial load of 4450 N. Estimate the L_{10} life of the bearing using the Standard method.

Problem Solution



Ex.(2.10) D = 10 mm (0.3937 in.)
l = 9.6 mm (0.378 in.)
Z = 14 rollers
i = 1 row of rollers

Ex.(11.6) $\gamma = 0.1538$

Table 11.16 at $\gamma = 0.1538$, $f_{cm} = 97.15$

Ex.(11.176) $$C = f_{cm}(il\cos\alpha)^{7/9} Z^{3/4} D^{29/27}$$

$$C = 97.15(1 \cdot 9.6 \cdot \cos 0°)^{7/9}(14)^{3/4}(10)^{29/27} = 48430N$$

$$F_e = F_r = 4450N$$

Ex.(11.167) $$L = \left(\frac{C}{F_e}\right)^{10/3} = \left(\frac{48430}{4450}\right)^{10/3} = 2856 \cdot 10^6\, revolutions$$

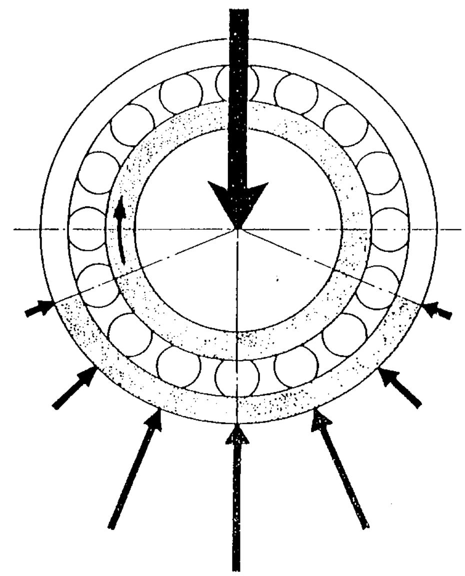
To compare this result of this standard method calculation with that of Ex. 11.6, it is necessary to introduce the b_m factor into the latter set of calculations. From Table 11.11, $b_m = 1.1$ for radial, cylindrical roller bearings. Hence, the bearing L_{10} life would be $(1.1)^4 \times 985 \times 10^6$ revolutions. It is apparent that the Standard method, which does not account for the precise internal load distribution provides only an approximate life estimate and in this case tends to overstate the bearing fatigue endurance capability.

Example 11.9 Standard Fatigue Life of a Radial, Spherical Roller Bearing

Problem Statement

The 22317 SRB of Ex. 7.8 experiences an outer raceway rotation of 900 rpm. Estimate the L_{10} life of the bearing using the Standard method.

Problem Solution



Ex.(2.7) D = 25 mm (0.9843 in.)
l = 20.76 mm (0.8154 in.)
$\alpha = 12°$
Z = 14 rollers per row
i = 2 rows of rollers

Ex.(2.9) $\gamma = 0.1810$

Table 11.16 at $\gamma = 0.1810$, $f_{cm} = 97.68$

Eq. (11.176) $C = f_{cm}(il\cos\alpha)^{7/9} Z^{3/4} D^{29/27}$

$$C = 97.68(2 \cdot 20.71 \cdot \cos 12°)^{7/9}(14)^{3/4}(25)^{29/27} = 399300N$$

$$\frac{F_a}{F_r} = \frac{22250}{89000} = 0.25$$

$$1.5\tan\alpha = 1.5\tan 12° = 0.3189$$

Since $F_a / F_r < 1.5\tan\alpha$, from Table 11.7 use $X = 1$ and $Y = 0.45\cot\alpha = 2.117$

Eq. (11.129)

$$F_e = XF_r + YF_a = 1 \cdot 89000 + 2.117 \cdot 22250 = 136100N$$

Eq. (11.160) $$L = \left(\frac{C}{F_e}\right)^{10/3} = \left(\frac{399300}{136000}\right)^{10/3} = 36.15 \cdot 10^6 \, revolutions$$

$$L = \frac{36.15 \cdot 10^6 \, revolutions}{900 \, revolutions / \min \cdot 60 \min / hr} = 669.5 hr$$

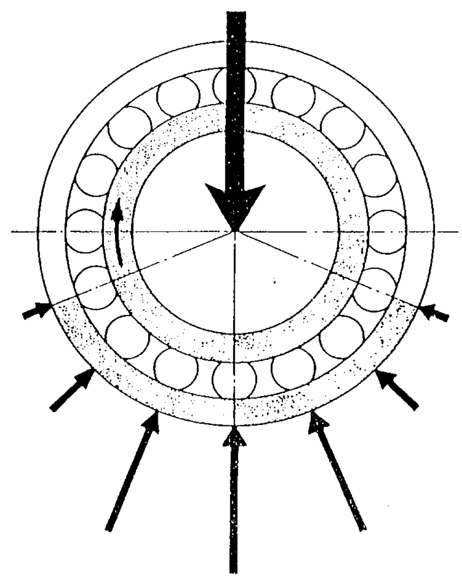
Example 11.10 Fatigue Life of a Radial, Spherical Roller Bearing Subjected to Variable Load

Problem Statement

The 22317 SRB of Ex. 11.9 experiences the repetitive loading cycle indicated in the table below while operating at shaft speed of 900 rpm. Estimate the L_{10} life of the bearing using the Standard method.

Condition	Time (min)	F_r (N)	F_r (lb)	F_a (N)	F_a (lb)
1	20	89,000	(20,000)	22,250	(5,000)
2	30	44,500	(10,000)	0	
3	10	22,250	(5,000)	22,250	(5,000)

Problem Solution



Ex. 11.9 $F_{e1} = 136{,}100$ N (30.950 lb)

$F_{e2} = 44{,}500$ N (10,000 lb)

Ex. 11.9 $1.5\tan\alpha = 0.3189$

$F_a / F_r > 1.5\tan\alpha$

Table 11.7 $X = 0.67$ and $Y = 0.67\cot\forall$

Ex. 2.7 $\alpha = 12°$

$$Y = 0.67\cot\alpha = 0.67\cot 12° = 3.152$$

Eq. (11.166)

$$F_{e3} = XF_r + YF_a = 0.67 \cdot 22{,}250 + 3.152 \cdot 22{,}250 = 85{,}040N(19{,}100lb)$$

Eq. (11.186) $$F_m = \left(\sum_{k=1}^{k=3} \frac{F_k^{10/3} N_k}{N} \right)^{3/10}$$

$$F_m = \left\{ \frac{20 \cdot (136{,}100)^{10/3} + 30 \cdot (44{,}500)^{10/3} + 10 \cdot (85{,}040)^{10/3}}{20 + 30 + 10} \right\}^{3/10} = 101{,}700 N (22{,}860 lb)$$

Ex. 11.9 C = 399,300 N (89.720 lb)

Eq. (11.180) $$L = \left(\frac{C}{F_m} \right)^{10/3} = \left(\frac{399{,}300}{101{,}700} \right)^{10/3} = 95.48 \cdot 10^6 \, revolutions$$

$$L = \frac{95.49 \cdot 10^6}{60 \min/hr \cdot 900 rpm} = 1768 hr$$

Example 11.11 Angular-Contact Ball Bearing Fatigue Life with 99% Reliability

Problem Statement

The 218 ACBB of Ex. 11.5 is operated at 3600 rpm under a 17,800 N (4000 lb) thrust load. Estimate the 99% reliability fatigue life of the bearing for inner ring rotation.

Problem Solution

Ex. 11.5 $L_{10} = 1770$ hr

Probability of Failure $\Im = 100 - 99 = 1\%$

Table 11.11 $b_m = 1.3$

$$Q_c = b_m Q_c = 1.3 Q_c$$

Eq. (11.70) and (11.75) $L_{10} = \left(\frac{Q_c}{Q_e}\right)^3 = \left(\frac{1.3Q_c}{Q_e}\right)^3 = 2.20 L_{10}$

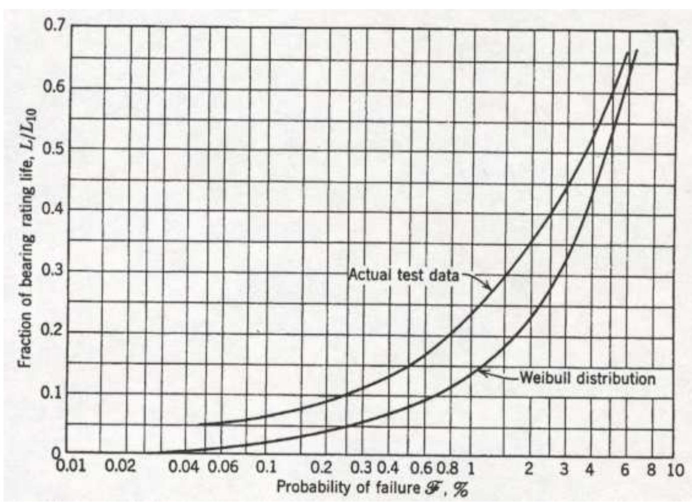


Fig. 11.31 **at $\Im = 1\%$, $L/L_{10} = 0.23$**

Therefore $L_1 = 0.23 \cdot 2.20 \cdot 1770 = 895.6hr$

Example 11.12 Deep-Groove Ball Bearing Fatigue Life with 95% Reliability

Problem Statement

The 209 DGBB of Ex. 11.7 achieved an estimated L_{10} life of 459.6 hr. Assuming contemporary steel and manufacturing processes, the bearing has a basic load rating C = 32,710 N (7350 lb). To achieve the same life with 95% reliability, what is the basic load rating of the required ball bearing?

Problem Solution

Probability of Failure $\Im = 100 - 95 = 5\%$

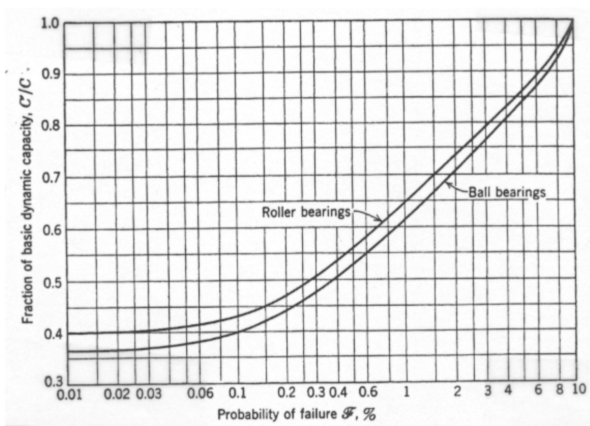


Fig. 11.32 **at $\Im$ = 5%, *C'/C* = 0.845**

Therefore, *C'* = 32,710/0.845 = 38,710 N (8699 lb)