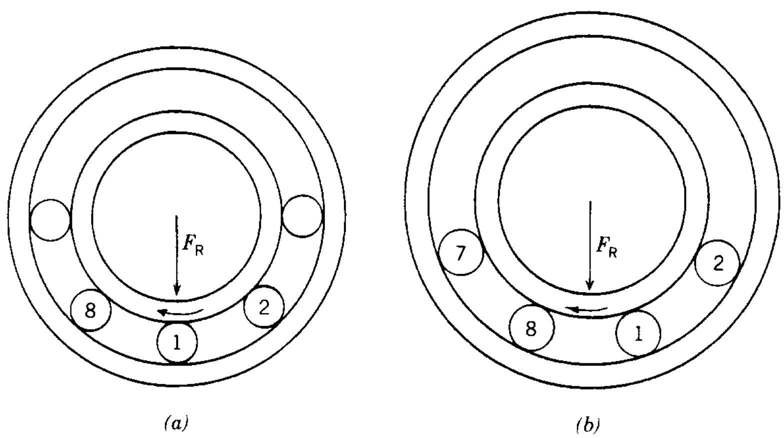
Example 14.1 Vibration of a Ball Bearing Due to Variable Elastic Compliance

Problem Statement

Consider a 204 radial ball bearing having a complement of 8, 7.938 mm (5/16 in.) diameter balls. The bearing supports a 4450 N (1000 lb) radial load. Determine the bearing deflections causing variable elastic compliance vibration.

Problem Solution

The illustration below (Fig. 14.3) shows the bearing ball set at two different times: (a) when ball no. 1 is directly under the applied load, and (b) when ball nos. 1 and 8 straddle the load symmetrically.



In (a), ball nos. 1, 2 & 8 carry the load; in (b) ball nos. 1, 2, 7 & 8 carry the load. Obviously, the bearing radial deflection is different in each situation. Using calculation methods from Chapter 7 (see also Chapter 1 in *Rolling Bearing Analysis, 5th Ed., Volume II*), the bearing radial deflection in situation (a) is estimated to be 0.04323 mm (0.001702 in.) and in (b) 0.04353 mm (0.001714 in.). The bearing deflection therefore varies approximately 0.0003 mm (12 µin.) causing a vibration having frequency equal to the cage rotational frequency multiplied by the number of balls. The frequency of this vibration occurs at the outer raceway rolling element pass frequency or

$$f_{bpor} = Zf_c \quad \textbf{where} \quad f_c = \frac{n_i}{2}\left(1 - \frac{D}{d_m}\cos\alpha\right)$$

Example 14.2 Raceway Waviness as a Bearing Vibration Source

Problem Statement

FIGURE CD14.1 Spherical roller bearing inner ring waviness, machine setup error. Each radial division equals 1 μm.

In Fig. CD14.1above the radial geometrical deviation of the surface from a true circle (shown on each trace as a dashed line) can be approximated as a function of

angular position. The shape is approximately sinusoidal; radial amplitude is then given by

$$r = A\sin\left(\frac{2\pi R\theta}{\lambda}\right) \quad \textbf{(CD14.1)}$$

where: ***A*** **= peak amplitude**
R **= circumferential distance measured from starting point θ = 0**
λ = wavelength of one cycle

$$\lambda = \frac{circumference}{waves} = \frac{2\pi R}{W} \quad \textbf{(CD14.2)}$$

From Eqs. (CD14.1) & (CD14.2)

$$r = A\sin(W\theta) \quad \textbf{(CD14.3)}$$

From Fig. CD14.1

$$r = A\sin(9\theta)$$

This type of discrete frequency waviness is an excitation source for vibration as well as a generator of dynamic force variations on bearing components. Defects of the magnitude shown in Fig. CD14.1 are rare and easily detected.

Waviness of relatively low number (usually odd) of waves per circumference occurs because of inaccuracies in grinding machine tooling or setup involving sliding contact shoe supports for the work-piece. Contact of two high points simultaneously on two shoes causes more material to be removed by the grinding wheel at a position opposite the shoes. Conversely, low points on the shoes result in less material being removed, producing high points on either side of the wheel work contact zone. This condition is detectable with conventional in-process gauging used to control diameter, provide the gauges are set up for 3-point diameter measurement as opposed to 2-point diameter measurement.

Example 14.3 Vibration of an Electric Motor Having a Lobed Shaft on Which a Bearing is Mounted

Problem Statement

Assembly of bearings in housings or on shafts with poorly controlled geometry can distort the bearing rings and raceways and produce wavy running surfaces that affect bearing vibration and noise. Fig. CD14.2 below illustrates a circumferential trace of a motor shaft-bearing journal on which a 6 mm bore ball bearing was mounted.

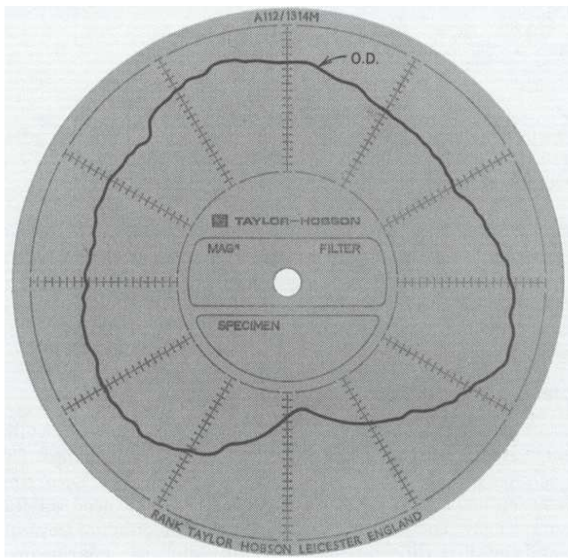


Fig. CD14.2

The motor was rejected after assembly for audible noise and for vibration as determined by hand-turning the armature while it was supported by the bearing. The shaft speed in this power tool application was 23000 rpm. The shaft 3-point out-of-roundness was approximately 24 µm (0.001 in.). Shaft diameter tolerance for this application was normally held within a total spread of one shaft to another of 8 µm (0.0003 in.). Fig. CD14.3 below shows traces of the bearing bore and ball groove after disassembly from the shaft. Both surfaces are less than 1 µm (0.00004 in.) out-of-round. Finally Fig. CD14.4 shows a trace of the ball groove as mounted on the shaft. This indicated that the raceway in the mounted condition exhibited 16 µm (0.0006 in.) of 3-point out-of-roundness. Note the two local imperfections on the raceway. The largest is approximately 2 µm (0.00008 in.) deep and is located on one of the lobes. This defect probably occurred during press-fit assembly of the bearing on the shaft or from damage during running.

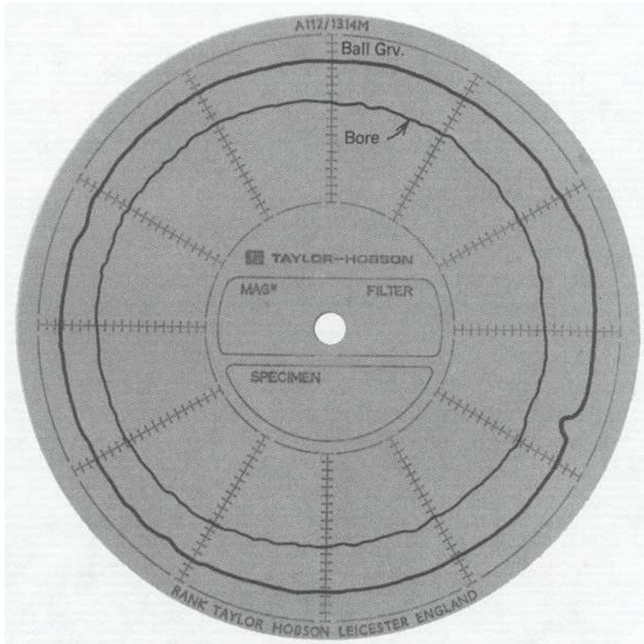


Fig. CD14.3 Inner raceway after disassembly

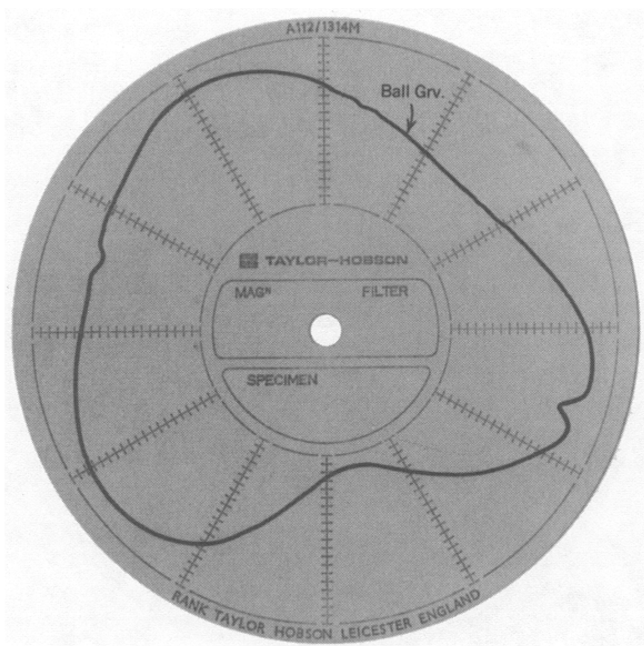


Fig. CD14.4 Inner raceway after mounting on shaft

The bearing has a complement of 6 balls; therefore, the 3 high points on the distorted inner raceway could be in contact with 3 balls, while the other 3 balls carry no load. This condition would decrease the bearing stiffness either axially or radially. Conversely, larger individual ball loads would be expected to be generated

during parts of the shaft rotational cycle, tending to increase stiffness while simultaneously causing large axial vibration.

Problem Solution

The cage operating speed was calculated using equations from Chapter 10 to be 138 Hz. The cage speed relative to the inner was calculated to be 245 Hz. Therefore, the rate at which a wave cycle passes a ball was $3 \times 245 = 735$ Hz. The rate at which any of the high points passes from one ball to the next equals the product of the cage speed relative to the inner raceway and the number of balls; i. e., $6 \times 245 = 1470$ Hz. This is harmonically related to the wave passage frequency, because the number of balls is a multiple of the number of waves. Accordingly, there is the potential for large-amplitude vibrations with two fundamental frequencies; i. e., 735 and 1470 Hz. These would be well within the audible range. Additionally, many high harmonics of each would be expected. These would have the potential for excitation of various structural resonances in the motor.

Example 14.4 Solving a Noise Problem UsingVibration Frequency Spectra

Problem Statement

Fig. CD14.5 below shows the sound frequency spectrum of a vacuum cleaner motor rejected after assembly for noise. The spectrum was obtained from airborne noise

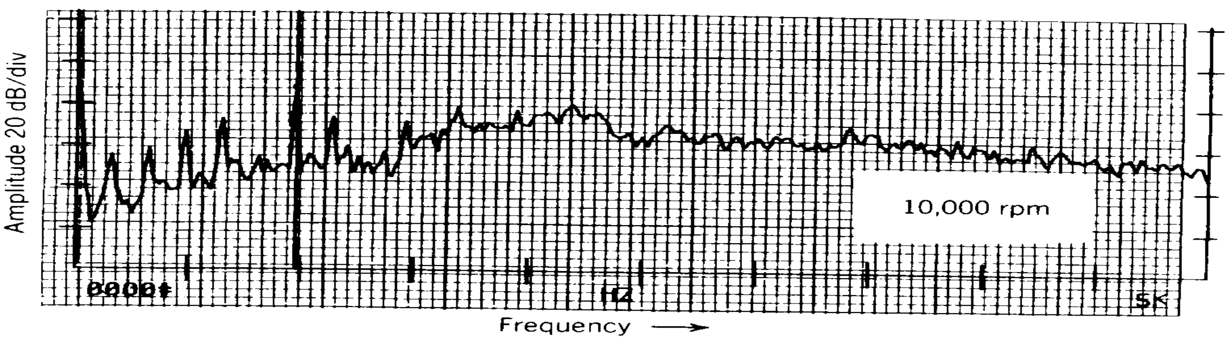


Fig. CD14.5 Electric motor sound spectrum (noise problem)

measurement using a microphone. The amplitude scale is logarithmic and not calibrated; each vertical division represents an amplitude increase of a factor of 10. Lower frequencies are attenuated as is common in sound measurement. The normal operating speed of the motor is 20,000 rpm. Neither the sound spectrum nor qualitative audible evaluation revealed anything abnormal at this speed. When the power to the motor was turned off and the motor was coasting down, however, a distinct rumble was heard at a speed estimated to be around 10,000 rpm. The spectrum above was then obtained with the motor running at that speed. It seemed likely that some system resonance was occurring as the motor coasted down and passed a critical speed.

Problem Solution

The spectrum shows distinct frequency peaks at 165.5, 325, 487.5, 650, 975 and 1137.5 Hz. Next, a spectrum was obtained from a similar, but deemed good quality, motor; this is shown by Fig. CD14.6 below. This spectrum shows a single peak at

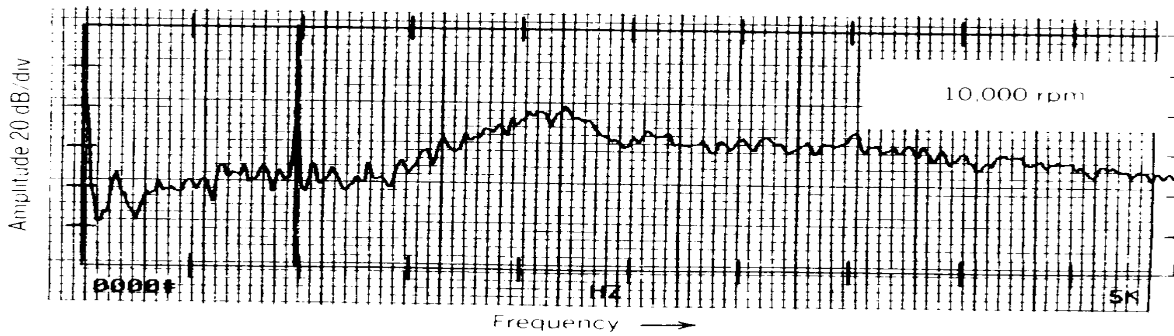


Fig. CD14.6 Electric motor sound spectrum (good quality motor)

975 Hz. This peak was expected to be the blade pass frequency on the 6 blade motor fan. This means that the actual running speed was 9750 rpm. In this case, the measured spectral peaks on the noisy motor were all harmonically related to the actual running speed. Harmonics of the running speed usually occur when mechanical looseness exists. This could be caused by a loose bearing mount.

Looseness would result in low stiffness, lower resonant frequency, and "play" in the system. The unbalance force, rotating at shaft speed, could then produce significant vibration amplitude at that frequency. The harmonics probably result from either of two effects: (1) directional stiffness vibration or (2) shocks occurring if the load zone shifts from one side of the bearing to the other in an unstable manner.

The noisy motor was disassembled. A spring clip, which is used to retain the bearing outer ring in a plastic housing had been improperly seated during assembly, resulting in a loose bearing mount. Repair and reassembly reduced the noise to an acceptable level. Diagnosis of this problem could have been accomplished using vibration transducers in lieu of sound measurement. The audible evaluation of the transient vibration during coast-down also provided a clue to the source of the problem.

Example 14.5 Solving an Electric Motor Bearing Noise Problem UsingVibration Frequency Spectra

Problem Statement

Fig. CD14.7 below shows the vibration frequency spectra two small electric motors on the same plot; motor shaft speed is 3600 rpm. These data were obtained from a

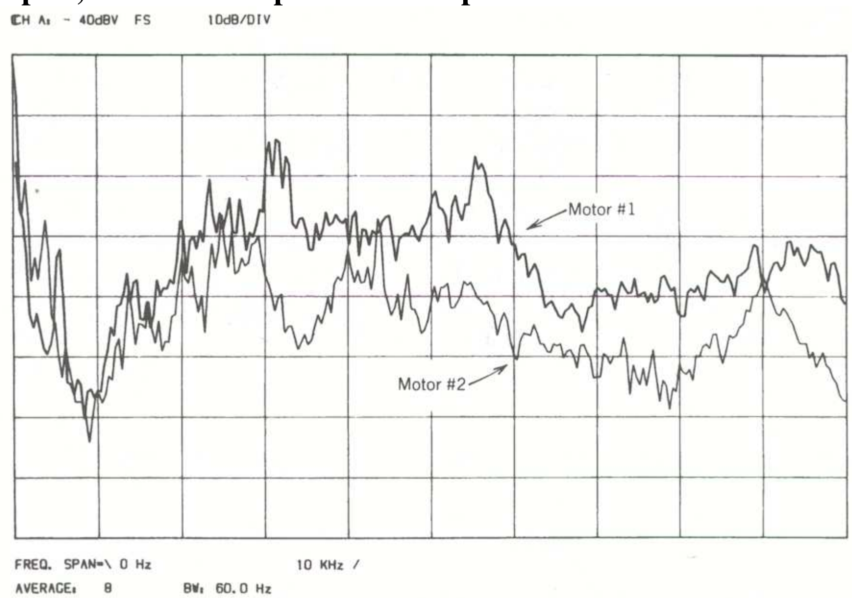


Fig. CD14.7 Electric motor vibration spectra

small accelerometer screwed into a nut glued to the end cover. The frequency range investigated was to 10000 Hz, covering the most important part of the audible frequency range at usual operating speeds.

Problem Solution

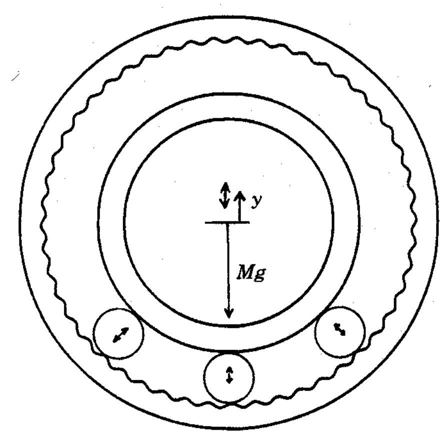
The rms vibration amplitude at each frequency is plotted in volts dB, where the dB value = 20 $\log_{10}$(voltage/reference voltage). In this case, reference voltage = 1. Each major division on the plot = 10 dB; -40 db (0.01 v) ≤ amplitude scale ≤ - 120 dB (10^{-6} v). The accelerometer output is 0.010 v/g (g = gravitational constant).

One motor is seen to have vibration amplitudes from 5 to 25 dB higher than the other motor over much of the frequency range. The motor having the higher vibration also had torque readings > 2x those of the other motor. Frequency spectra taken at various points on the motors gave the same results. Spectra in other frequency ranges were taken yielding no conclusive results regarding the origin of the vibration.

After disassembly of the motors, shafts and housings were checked for geometry and found normal; also, bearing torques were measured. The bearings from the motor with high vibration had rubbing seals; the other set had non-contacting seals. With seals removed, the four bearings were vibration tested separately. It was found that one of the bearings from the high vibration motor gave high reading on the bearing vibration tester. Spectrum analysis indicated harmonics of the outer raceway ball pass frequency. Examination of the outer raceway revealed a defect related to manufacturing; the defect had apparently escaped detection in final inspection vibration testing.

Example 14.6 Vibration of a Rolling Bearing Due to Outer Raceway Waviness

Problem Statement For a bearing having an outer raceway with 50 waves on the circumference, estimate the amplitude of the waviness that could cause sufficient acceleration to momentarily cause the bearing to be unloaded if the shaft speed is 1800 rpm (30 Hz) and the cage speed is 11 rps.



Problem Solution

The rate at which any ball passes over a wave cycle is the product of the cage speed and the number of waves per circumference of the outer raceway; in this case 50 waves × 11 rps = 550 wave cycles/sec.

The bearing will unload under the condition:

$$Mg = MA(2\pi f)^2$$

or amplitude

$$A = \frac{g}{(2\pi f)^2} = \frac{980 mm/\sec^2}{(2\pi \cdot 550 cycles/\sec)^2} = 8.208 \cdot 10^{-4} mm (3.321 \cdot 10^{-5} in.)$$

Note that wave amplitude is usually expressed in terms of peak-to-valley units of micrometers (10^{-6} m or µm), in this case 1.64 µm.

Example 14.7 Roller Waviness as a Bearing Vibration Source

Problem Statement

A cylinder roller rotated in a waviness-testing machine exhibited the waviness trace shown by Fig. CD14.8 below.

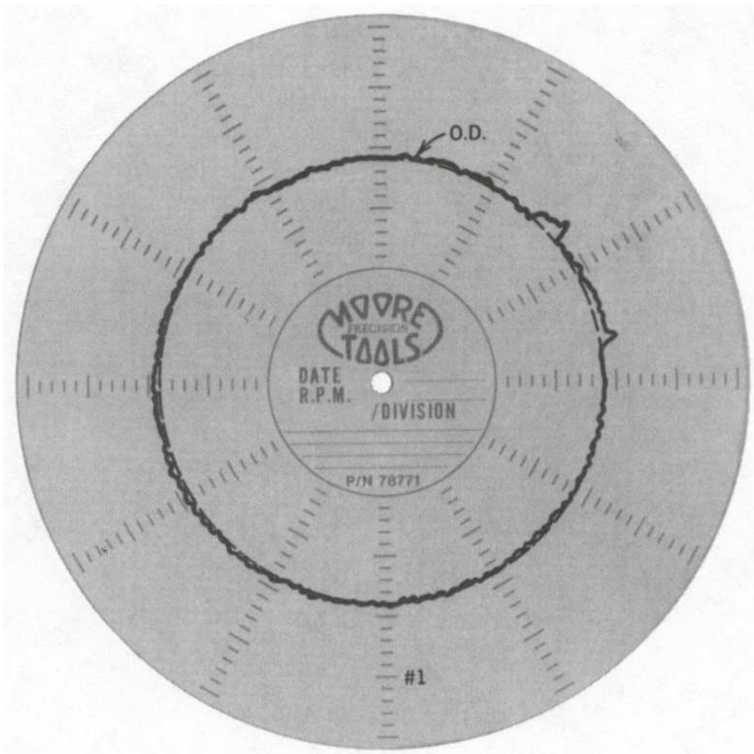


FIGURE CD14.8 Low amplitude, high frequency waviness

The amplified output of the velocity transducer, analyzed according to frequency spectrum, is shown by Fig. CD14.9 below.

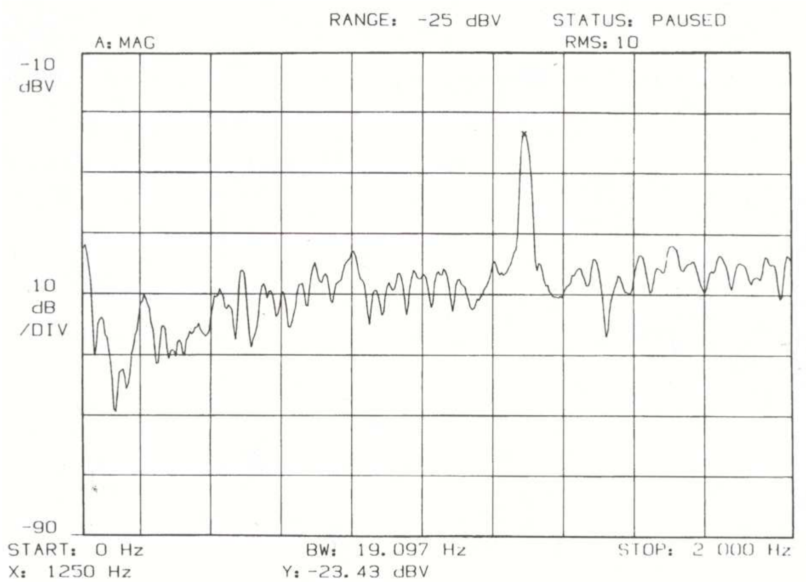


FIGURE CD14.9 Roller waviness velocity spectrum

The abscissa covers a frequency range of 0 – 2000 Hz. The roller was tested at 720 rpm (12 Hz); therefore, this frequency range would detect a dominant wave pattern up to 2000/12 = 166 waves/circumference. The ordinate gives rms voltage in dB referenced to 1 v, ranging from –10 dB full scale (0.3162 v) to –90 dB (3.162 A10^{-5}v). Nominal calibration for the velocity transducer and amplifier is 3.0 µv/µin.-sec. A cursor mark is indicated at the peak, with corresponding coordinates printed below the frequency axis. The peak occurs at 1250 Hz; rms amplitude = -23.48 dB. Since roller test speed = 12 Hz, the frequency at which the peak occurs corresponds approximately to 104 waves/circumference.

For the above data, determine (1) the rms radial velocity of the predominant waviness, (2) the number of waves per circumference, and (3) its average peak-to-valley amplitude in :m.

Problem Solution

The rms voltage is determined from

$$-23.43dB = 20\log_{10} V$$

Therefore: $V = 0.06738$

rms velocity is determined by dividing the voltage by the transducer conversion of 3 µV/µin.×sec. Therefore:

$$\left|\frac{dr}{dt}\right|_{rms} = (22{,}460\mu in./\sec)570\mu m/\sec$$

In Eq. (CD14.3), the radial deviation *r* is given as a function of angular position θ on the component and the number waves *W*.

In the waviness test apparatus, the part is rotated; therefore, the radial deviation *r* is a function of time. Any angular location on the part also becomes a function of time with respect to the fixed location of the transducer used to measure the radial deviations; hence:

$$\theta = 2\pi Nt \qquad \textbf{(CD14.4)}$$

where: N = test rotational speed (rps). Therefore:

$$W\theta = 2\pi NtW \qquad \textbf{(CD14.5)}$$

and

$$r = A\sin(2\pi NtW) \qquad \textbf{(CD14.6)}$$

The measured radial velocity is the change of the radial deviation with time.

$$\dot{r} = 2\pi NWA\cos(2\pi NtW) \qquad \textbf{(CD14.7)}$$

and

$$\dot{r}_{rms} = 1.414 \cdot NWA \qquad \textbf{(CD14.8)}$$

From the measured rms velocity, the peak amplitude *A* can be estimated.

$$22460 = 1.414 \cdot \pi \cdot \left(\frac{720}{60}\right) \cdot 104 \cdot A$$

***A* = 0.1029 μm (4.05 × 10^{-6} in.)**

Peak-to-valley amplitude = 2A = 0.206 (8.1×10^{-6} in.)

Example 14.8 Determination of Cage and Ball Rotational Frequencies and Waviness Orders for a Ball Bearing

Problem Statement

A 203 angular-contact ball bearing operating at 1800 rpm shaft speed has the following internal dimensions:

D	=	**6.747 mm (0.2656 in.)**
d_m	=	**28.5 mm (1.122 in.)**
α	=	**12°**

Calculate the cage and ball rotational frequencies. Also, estimate the waviness orders for each component that fall within vibration testing bands.

Problem Solution

n_i **= 1800 rpm = 30 rps = 30 Hz**

Eq. (14.6)

$$f_c = \frac{n_i}{2}\left(1 - \frac{D}{d_m}\cos\alpha\right) = \frac{30}{2}\left(1 - \frac{6.747\cdot\cos(12)}{28.5}\right) = 11.53\,Hz$$

Eq. (14.7)

$$f_{ci} = \frac{n_i}{2}\left(1 + \frac{D}{d_m}\cos\alpha\right) = \frac{30}{2}\left(1 + \frac{6.747\cdot\cos(12)}{28.5}\right) = 18.47\,Hz$$

Eq. (14.10)

$$f_R = \frac{n_i d_m}{2D}\left[1 - \left(\frac{D}{d_m}\cos\alpha\right)^2\right] = \frac{30\cdot 28.5}{2\cdot 6.747}\left[1 - \left(\frac{6.747\cdot\cos(12)}{28.5}\right)^2\right] = 59.97\,Hz$$

Therefore, waviness orders of the components fall within the vibration test bands approximately as indicated in Table 14.1 below.

Table 14.1 Waviness Orders Within Vibration Test Bands

Component	50-300 Hz	300-1800 Hz	1800-10000 Hz
outer raceway	4-26	4-26	4-26
inner raceway	2*-16	17-97	97-541
balls	2-15	6-30	31-167

***including 2-point & 3-point out-of-roundness**

Example 14.9 Effect of Outer Raceway Damage on Ball Bearing Vibration

Problem Statement

A rig testing two 205 ball bearings, mounted in pillow block at opposite ends of a belt-driven shaft was operated at 1690 rpm. The outer raceway of one bearing contained damage, approximately 1.6 mm (0.0625 in.) diameter, located in the bottom of the groove and in the load zone. The bearing internal dimensions are:

D	=	**7.938 mm (0.3125 in.)**
d_m	=	**39.04 mm (1.537 in.)**
Z	=	**9 balls**

Fig. CD14.10 shows vibration frequency spectra of the two bearings. Frequency span is 0-10000 Hz and full-scale amplitude is –20 dB. Data were obtained with a stud-mounted accelerometer, 0.010 v/g.

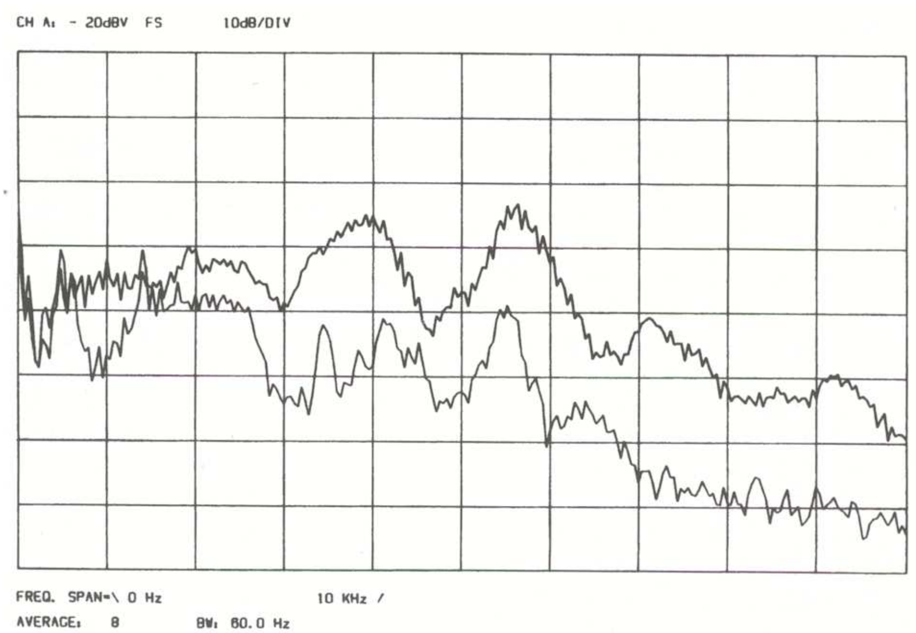


FIGURE CD14.10 Vibration of damaged and undamaged bearings

Problem Analysis

The vibration amplitudes of the damaged bearing are 20 dB greater than those of the undamaged bearing at most frequencies above 3000 Hz. The calculated bearing frequencies are:

$$n_i = \frac{1690}{60} = 28.2Hz$$

Eq. (14.6)

$$f_c = \frac{n_i}{2}\left(1 - \frac{D}{d_m}\cos\alpha\right) = \frac{28.2}{2}\left(1 - \frac{7.938\cdot\cos(0)}{39.04}\right) = 11.23Hz$$

Eq. (14.8) $$f_{REpor} = Zf_c = 9\cdot 11.23 = 101.1Hz$$

Eq. (14.7)

$$f_{ci} = \frac{n_i}{2}\left(1 + \frac{D}{d_m}\cos\alpha\right) = \frac{28.2}{2}\left(1 + \frac{7.938\cdot\cos(0)}{39.04}\right) = 16.97Hz$$

Eq. (14.9) $$f_{REpir} = Zf_{ci} = 9\cdot 16.97 = 152.7Hz$$

The vibration frequency spectrum of the damaged bearing shows about 10 peaks in each 1000 segment whose spacing corresponds to Zf_c. The figure illustrates the major effect of local bearing damage on vibration in the higher-frequency regions.

Fig. CD14.11 below was taken on the damaged bearing over 2500 Hz.

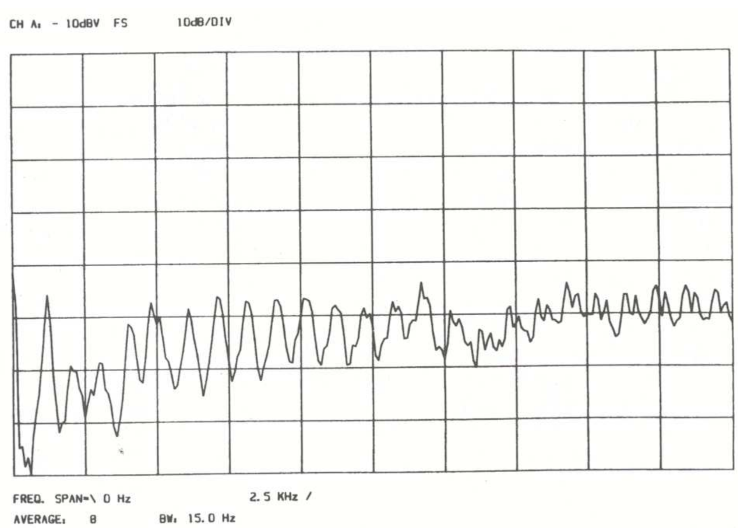


FIGURE CD14.11 Vibration of damaged bearing.

Fig. CD14.11 clearly shows harmonics of the rolling element-pass frequency over the outer raceway from 500 to 1250 Hz. Amplitudes of harmonics from 700-1200 Hz were approximately 10 dB greater than vibration amplitudes of the undamaged bearing in this range. Below 700 Hz, amplitudes of the two bearings were the same.

Depending on the presence of other sources of machine vibration and the magnitude of bearing damage, it might also be possible to successfully identify a problem at low frequencies.