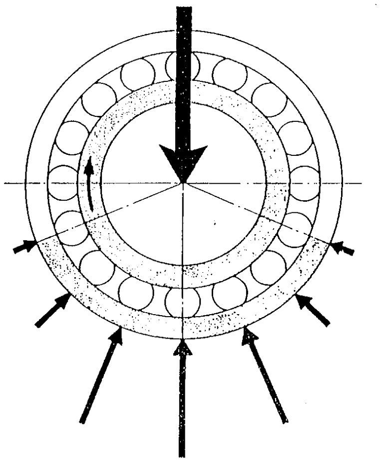
Example 7.1 Internal Distribution of Applied Radial Load in a Radial Ball Bearing Having Specified Clearance

Problem Statement



The 209 DGBB of Ex. 2.1 supports a radial load of 8900 N. Determine the ball-raceway load at each ball location.

Problem Solution

Ex. 2.1

$Z = 9$ balls
$P_d = 0.0150$ mm (0.0006 in.)
$d_m = 65$ mm (2.559 in.)
$D = 12.7$ mm (0.50 in.)

Ex. 2.2

$f_i = f_o = 0.52$

Ex. 2.5

$\gamma = 0.1954$
$\Sigma\rho_i = 0.202\ \text{mm}^{-1}$ (5.126 in.$^{-1}$)
$F(\rho)_i = 0.9399$
$\Gamma\rho_o = 0.138\ \text{mm}^{-1}$ (3.500 in.$^{-1}$)
$F(\rho)_o = 0.9120$

Fig. 6.4 $\delta_i^* = 0.602$ and $\delta_o^* = 0.658$

Eq. (7.8)

$$K_{pi} = 2.15\cdot 10^5 \Sigma\rho_i^{-1/2}\left(\delta_i^*\right)^{-3/2} = 2.15\cdot 10^5\cdot(0.202)^{-1/2}(0.602)^{-3/2} = 1.026\cdot 10^6\ N/mm^{1.5}$$

Eq. (7.8)

$$K_{po} = 2.15\cdot 10^5 \Sigma\rho_o^{-1/2}\left(\delta_o^*\right)^{-3/2} = 2.15\cdot 10^5\cdot(0.138)^{-1/2}(0.658)^{-3/2} = 1.089\cdot 10^6\ N/mm^{1.5}$$

Eq. (7.6)

$$K_n = \left(K_{pi}^{-1/1.5} + K_{po}^{-1/1.5}\right)^{-1.5} = \left(1.026^{-0.667} + 1.089^{-0.667}\right)^{-1.5}\cdot 10^6 = 3.735\cdot 10^5\ N/mm^{1.5}$$

Eq. (7.22)

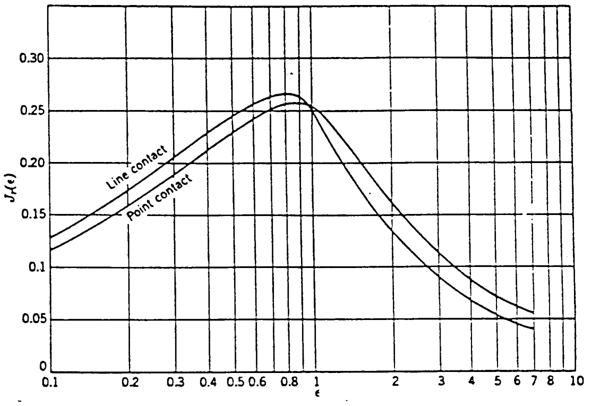
$$F_r = ZK_n\left(\delta_r - \frac{1}{2}P_d\right)^{1.5} J_r(\varepsilon)$$

$$(1)\,8900 = 9\cdot 3.735\cdot 10^5\left(\delta_r - \frac{0.0150}{2}\right)^{1.5} J_r(\varepsilon)$$

Eq. (7.12) $$\varepsilon = \frac{1}{2}\left(1 - \frac{P_d}{2\delta_r}\right)^{1.5}$$

$$(2)\varepsilon = \frac{1}{2}\left(1 - \frac{0.0150}{2\delta_r}\right)^{1.5} = 0.5 - \frac{0.00375}{\delta_r}$$

Use Fig. 7.2 and Eq. (1) and (2) to solve for δ_r



$\delta_r = 0.06041$ mm (0.06041 in.), $\varepsilon = 0.438$ and $J_r(0.438) = 0.218$

Eq. (7.19) $$F_r = ZQ_{\max}J_r(\varepsilon)$$

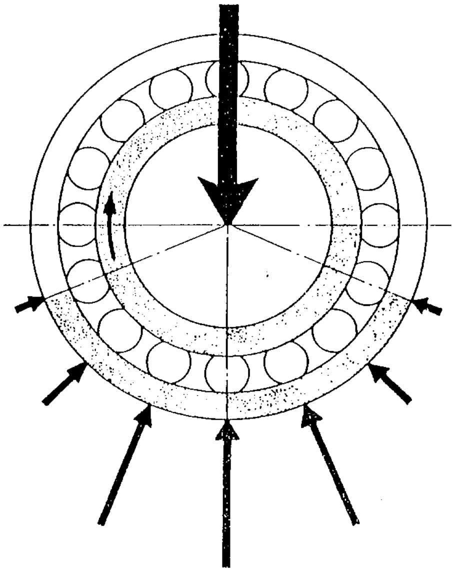
$$Q_{\max} = \frac{F_r}{ZJ_r(\varepsilon)} = \frac{8900}{9 \cdot 0.218} = 4536N$$

Eq. (7.15) $$Q_\psi = Q_{\max}\left[1 - \frac{1}{2\varepsilon}(1 - \cos\psi)\right]^{1.5}$$

$$Q_\psi = 4536\left[1 - \frac{1}{2 \cdot 0.438}(1 - \cos\psi)\right]^{1.5} = 4536(1.142\cos\psi - 0.142)^{1.5}$$

ψ	$\cos\psi$	Q_ψ N (lb)
0	1	4536 (1019)
±40°	0.7660	2846 (638.6)
±80°	0.1737	61 (13.7)
±120°	-0.5000	0
±160°	-0.9397	0

Example 7.2 Internal Distribution of Applied Radial Load in a Radial Ball Bearing Having Zero Clearance



Problem Statement

Use Stribeck's equation to determine the internal load distribution for the 209 DGBB of Ex. 7.1

Problem Solution

Eq. (7.23) $$Q_{\max}=\frac{4.37F_r}{Z\cos\alpha}=\frac{4.37\cdot 8900}{9\cos 0^\circ}=4321N$$

Eq. (7.15) $$Q_\psi=Q_{\max}\left[1-\frac{1}{2\varepsilon}(1-\cos\psi)\right]^{1.5}$$

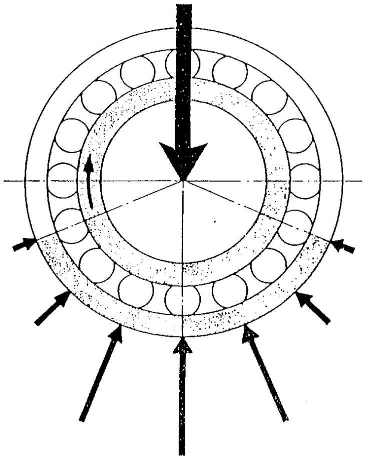
For Eq. (7.23), $\varepsilon=0.5$

$$Q_\psi=4321\left[1-\frac{1}{2\cdot 0.5}(1-\cos\psi)\right]^{1.5}=4321\cos^{1.5}\psi$$

ψ	$\cos\psi$	Q_ψN (lb)
0	1	4321 (971.2)
±40°	0.7660	2897 (650.7)
±80°	0.1737	313 (70.1)
±120°	-0.5000	0
±160°	-0.9397	0

Example 7.3 Internal Distribution of Applied Radial Load in a Radial Cylindrical Roller Bearing Having Specified Clearance

Problem Statement



The 209 CRB of Ex. 2.7 supports a radial load of 4450 N (1000 lb). Determine the roller-raceway load at each roller location and the extent of the load zone.

Problem Solution

Ex. 2.3

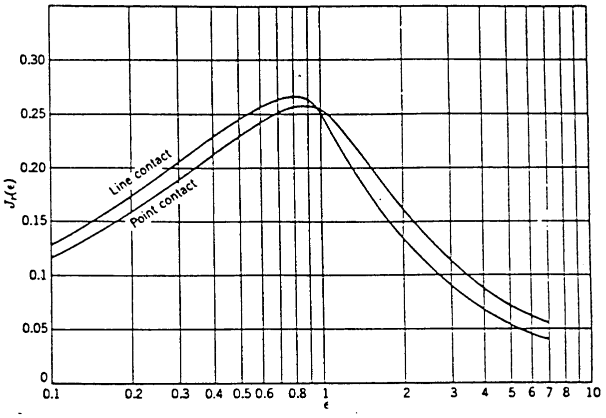
$Z = 14$ rollers
$P_d = 0.041$ mm (0.0016 in.)
$d_m = 65$ mm (2.559 in.)
$l = 9.6$ mm (0.3780 in.)

Eq. (7.9) $$K_l = 7.86 \cdot 10^4 l^{8/9} = 7.86 \cdot 10^4 \cdot (9.6)^{8/9} = 5.869 \cdot 10^5 \, N/mm^{10/9}$$

Eq. (7.6) $$K_n = \left(K_l^{-0.9} + K_l^{-0.9}\right)^{-10/9} = \left(5.869^{-0.9} + 5.869^{-0.9}\right)^{-10/9} \cdot 10^5 = 2.720 \cdot 10^5 \, N/mm^{1.11}$$

Eq. (7.22) $$F_r = ZK_n\left(\delta_r - \frac{1}{2}P_d\right)^{10/9} J_r(\varepsilon)$$

$$(1) 4450 = 14 \cdot 2.72 \cdot 10^5\left(\delta_r - \frac{0.041}{2}\right)^{10/9} J_r(\varepsilon)$$



Eq. (7.12) $$\varepsilon = \frac{1}{2}\left(1 - \frac{P_d}{2\delta_r}\right)^{1.5}$$

$$(2)\varepsilon = \frac{1}{2}\left(1 - \frac{0.041}{2\delta_r}\right)^{1.5} = 0.5 - \frac{0.01025}{\delta_r}$$

Use Fig. 7.2 and Eq. (1) and (2) to solve for δ_r

$\delta_r = 0.0320$ mm (0.00126 in.), $\varepsilon = 0.1824$ and $J_r(0.1824) = 0.165$

Eq. (7.19) $F_r = ZQ_{\max} J_r(\varepsilon)$

$$Q_{\max} = \frac{F_r}{ZJ_r(\varepsilon)} = \frac{4450}{14 \cdot 0.165} = 1926N$$

Eq. (7.15) $Q_\psi = Q_{\max}\left[1 - \frac{1}{2\varepsilon}(1-\cos\psi)\right]^{10/9}$

$$Q_\psi = 1926\left[1 - \frac{1}{2 \cdot 0.1824}(1-\cos\psi)\right]^{10/9} = 1926(2.741\cos\psi - 1.741)^{10/9}$$

ψ	$\cos\psi$	Q_ψ N (lb)
0°	1	1926 (432.8)
±25.71°	0.9010	1355 (304.7)
±51.42°	0.6237	0
±77.13°	0.2227	0
±102.84°	-0.2227	0
±128.55°	-0.6237	0
±154.26°	-0.9010	0
180°	-1	0

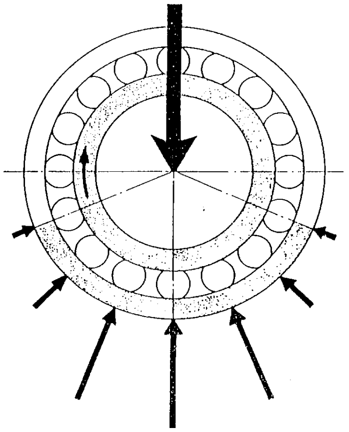
Eq. (7.15)

$$\psi_l = \cos^{-1}\left(\frac{P_d}{2\delta_r}\right)$$

$$\psi_l = \cos^{-1}\left(\frac{0.041}{2\cdot 0.0320}\right) = \pm 50.17^\circ$$

Example 7.4 Internal Distribution of Applied Radial Load in a Radial Cylindrical Roller Bearing Having Nominal Clearance

Problem Statement



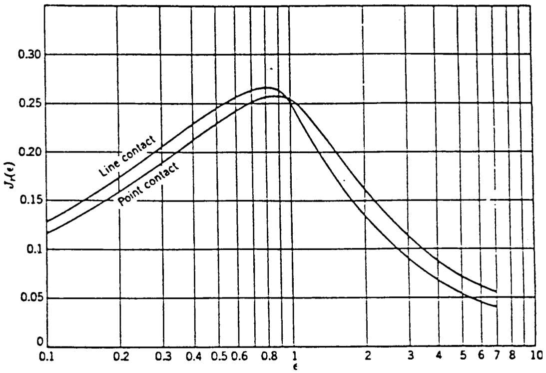
Using Eq. (7.24) to determine $Q_{\max}$, evaluate the load distribution which occurs for the 209 CRB of Ex. 7.3.

Problem Solution

Eq. (7.24) $$Q_{\max} = \frac{5F_r}{Z\cos\alpha} = \frac{5 \cdot 4450}{14\cos 0°} = 1589N$$

Eq. (7.19) $$J_r(\varepsilon) = \frac{F_r}{ZQ_{\max}} = \frac{4450}{14 \cdot 1589} = 0.2000$$

Fig. (7.2) $\varepsilon = 0.28$



Eq. (7.15) $$Q_\psi = Q_{\max}\left[1 - \frac{1}{2\varepsilon}(1-\cos\psi)\right]^{10/9}$$

$$Q_\psi = 1589\left[1 - \frac{1}{2 \cdot 0.28}(1-\cos\psi)\right]^{10/9} = 1589(1.786\cos\psi - 0.786)^{10/9}$$

Eq. (7.13) $$\psi_l = \cos^{-1}\left(\frac{P_d}{2\delta_r}\right)$$

Eq. (7.12) $$\varepsilon = \frac{1}{2}\left(1 - \frac{P_d}{2\delta_r}\right)^{1.5}$$

Therefore

$$\psi_l = \cos^{-1}(1-2\varepsilon) = \cos^{-1}(1-2 \cdot 0.28) = \pm 63.9°$$

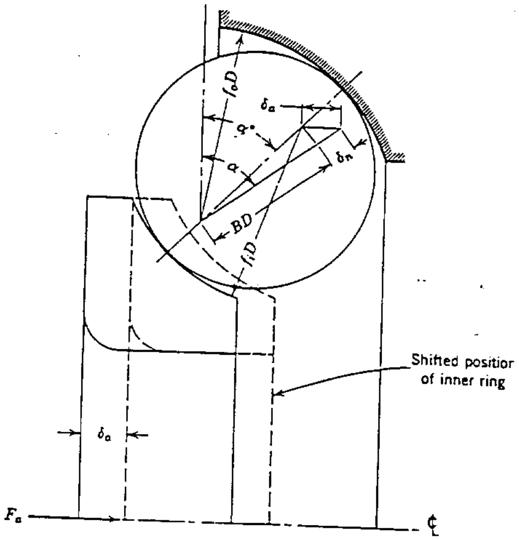
ψ	$\cos\psi$	Q_ψ N (lb)
0°	1	1589 (357.1)
±25.71°	0.9010	1280 (287.6)
±51.42°	0.6237	461 (103.6)
±77.13°	0.2227	0
±102.84°	-0.2227	0
±128.55°	-0.6237	0
±154.26°	-0.9010	0
180°	-1	0

Example 7.5 Increase of Ball-Raceway Contact Angle with Applied Thrust Load in an Angular-Contact Ball Bearing

Problem Statement

The 218 ACBB of Ex. 2.3 supports a statically applied thrust load of 17800 N. Considering a ball complement of 16, determine the:

- **ball-raceway contact angle**
- **ball-raceway normal ball load**
- **bearing axial deflection**



Problem Solution

Ex. (2.3) $B = 0.0464$
$\alpha^0 = 40°$
$D = 22.23$ mm (0.875 in.)

Eq. (7.33)

$$\frac{F_a}{ZD^2K} = \sin\alpha\left(\frac{\cos\alpha^0}{\cos\alpha} - 1\right)^{1.5}$$

Fig. (7.5) At $B = 0.0464$, $K = 896.7$ MPa

$$\frac{F_a}{ZD^2K} = \frac{17800}{16\cdot(22.23)^2\cdot 896.7} = 0.002512 = \sin\alpha\left(\frac{\cos 40°}{\cos\alpha} - 1\right)^{1.5}$$

Eq. (7.34)

$$\alpha' = \alpha + \frac{\dfrac{F_a}{ZD^2K} - \sin\alpha\left(\dfrac{\cos\alpha^0}{\cos\alpha} - 1\right)^{1.5}}{\cos\alpha\left(\dfrac{\cos\alpha^0}{\cos\alpha} - 1\right)^{1.5} + 1.5\tan^2\alpha\left(\dfrac{\cos\alpha^0}{\cos\alpha} - 1\right)^{0.5}\cos\alpha^0}$$

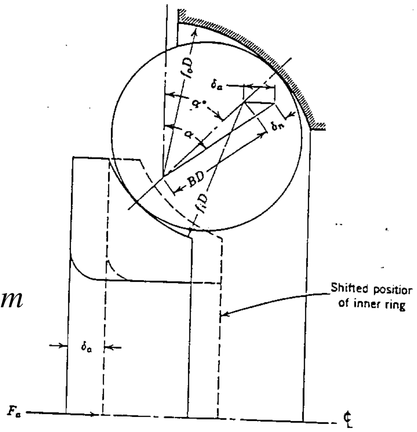
$$\alpha' = \alpha + \frac{0.002512 - \sin\alpha\left(\dfrac{\cos 40^0}{\cos\alpha} - 1\right)^{1.5}}{\cos\alpha\left(\dfrac{\cos 40^0}{\cos\alpha} - 1\right)^{1.5} + 1.5\tan^2\alpha\left(\dfrac{\cos 40^0}{\cos\alpha} - 1\right)^{0.5}\cos 40^0}$$

Eq. (7.34) is solved by assuming values of α. Iteration continues until absolute value α' - α approaches 0. $\alpha = 0.7260$ radians = 41.6°

Eq. (7.26) $Q = \frac{F_a}{Z \sin \alpha} = \frac{17800}{16 \sin 41.6°} = 1676N$

Eq. (7.36)

$$\delta_a = \frac{BD \sin\left(\alpha - \alpha^0\right)}{\cos \alpha} = \frac{0.0464 \cdot 22.23 \cdot \sin\left(41.6° - 40°\right)}{\cos 41.6°} = 0.0386mm$$



Example 7.6 Distribution of Load Among the Balls in an Angular-Contact Ball Bearing Subjected to Thrust Load Applied Eccentrically

Problem Statement

The 218 ACBB of Ex. 7.5 supports a static thrust load of 17800 N applied 50.8 mm from the bearing axis. Considering the ball-raceway contact angle remains constant at 41.6°, determine the magnitude of the maximum ball load and the extent of the load zone.

Problem Solution

$$\frac{2e}{d_m} = \frac{2 \cdot 50.8}{125.3} = 0.8110$$

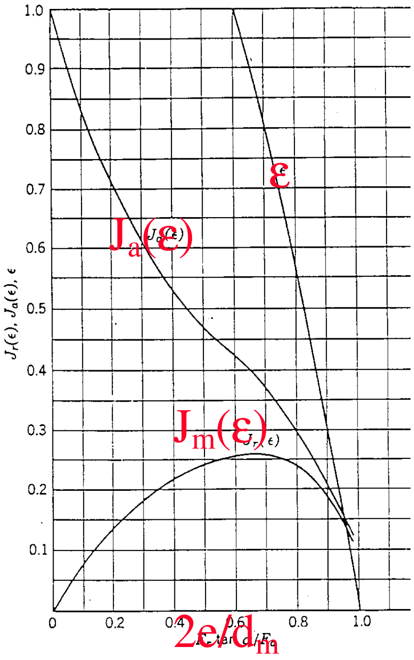


Fig. 7.8 $J_a = 0.285$, $J_m = 0.233$ and $\varepsilon = 0.525$

Eq. (7.45) $F_a = ZQ_{\max} J_a(\varepsilon)\sin\alpha$

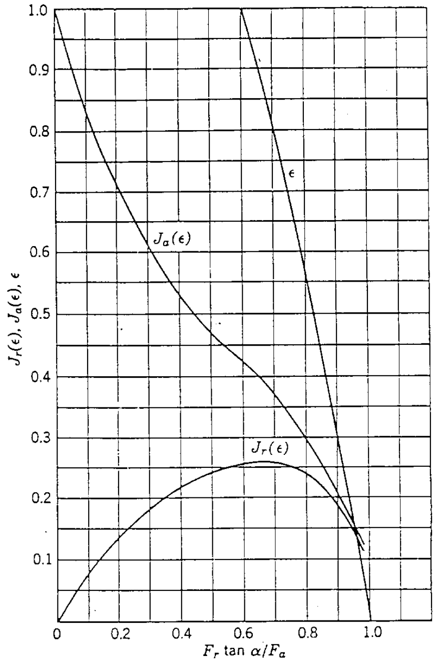
$$Q_{\max} = \frac{F_a}{ZJ_a(\varepsilon)\sin\alpha} = \frac{17800}{16 \cdot 0.285 \cdot \sin 41.6°} = 5878N$$

Eq. (7.40) $\psi_l = \cos^{-1}(1-2\varepsilon) = \cos^{-1}(1-2\cdot 0.525) = \pm 92.87°$

Example 7.7 Distribution of Load Among the Balls in an Angular-Contact Ball Bearing Subjected to Radial and Thrust Loading Applied in Combination

Problem Statement

The 218 ACBB of Ex. 7.5 supports a static thrust load of 17800 N combined with a 17800 radial load.. Considering the contact angle is constant at 40°, determine the loading on each ball and the extent of the load zone.



Problem Solution

$$\frac{F_r \tan \alpha}{F_a} = \frac{17800 \tan 40°}{17800} = 0.8391$$

Fig. 7.14 $J_a = 0.263$, $J_r = 0.221$ and $\varepsilon = 0.455$

Ex. (7.5) $Z = 16$ balls

Eq. (7.70)

$$Q_{\max} = \frac{F_a}{Z J_r(\varepsilon)\cos\alpha} = \frac{17800}{16 \cdot 0.221 \cos 40°} = 6571N$$

Eq. (7.62) $Q_\psi = Q_{\max}\left[1 - \frac{1}{2\varepsilon}(1-\cos\psi)\right]^{1.5}$

$$Q_\psi = 6571\left[1 - \frac{1}{2 \cdot 0.455}(1-\cos\psi)\right]^{1.5} = 6571(1.099\cos\psi - 0.0.0989)^{1.5}$$

Eq. (7.40) $\psi_l = \cos^{-1}(1-2\varepsilon) = \cos^{-1}(1-2\cdot 0.455) = \pm 84.78°$

ψ(°)	cos ψ	Q_ψ N (lb)
0	1	6571 (1477)
±22.5	0.9239	5765 (1296)
±45	0.7071	3670 (824.2)
±67.5	0.3827	1200 (269.6)
±90	0	0
±112.5	-0.3827	0
±135	-0.7071	0
±157.5	-0.9239	0
180	-1	0

Example 7.8 Distribution of Load Among the Rollers in a Double-Row Spherical Roller Bearing Subjected to Radial and Thrust Loading Applied in Combination

Problem Statement

The 22317 SRB of Ex. 2.7 supports a static thrust load of 22250 N combined with a 89000 radial load. Estimate the loading on each rollerand the extent of the load zone for each row of rollers,

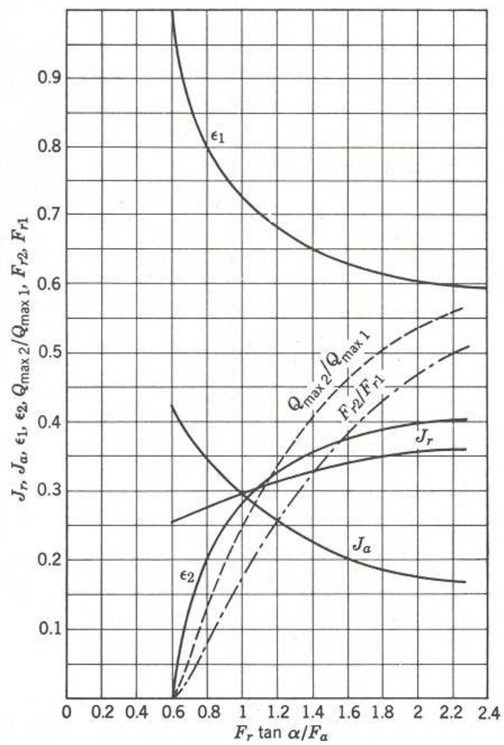
Problem Solution

Ex. (2.8) α = 12E

Ex. (2.8) $Z = 14$ rollers per row

$$\frac{F_r \tan\alpha}{F_a} = \frac{89000 \tan 12°}{22250} = 0.8502$$

Fig. (7.17)



$J_r = 0.303$, $J_a = 0.370$, $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.2$ and $Q_{max2}/ Q_{max1}=0.220$

Eq. (7.80) $F_r = Z Q_{\max 1} J_r(\varepsilon_1)\cos\alpha$

$$Q_{\max 1} = \frac{89000}{14 \cdot 0.303 \cos 12°} = 21450N$$

$$Q_{\max 2} = \frac{Q_{\max 2}}{Q_{\max 1}} Q_{\max 1} = 0.220 \cdot 21450 = 4719N$$

Eq. (7.62) $Q_{\psi 1} = Q_{\max 1}\left[1 - \frac{1}{2\varepsilon_1}(1 - \cos\psi)\right]^{1.11}$

$$Q_{\psi 1}=21450\left[1-\frac{1}{2\cdot 0.8}(1-\cos\psi)\right]^{1.11}=21450(0.625\cos\psi-0.375)^{1.11}$$

Eq. (7.40) $\psi_{l1}=\cos^{-1}(1-2\varepsilon_1)=\cos^{-1}(1-2\cdot 0.8)=\pm 126.87°$

Eq. (7.62) $Q_{\psi 2}=Q_{\max 2}\left[1-\frac{1}{2\varepsilon_2}(1-\cos\psi)\right]^{1.11}$

$$Q_{\psi 2}=4719\left[1-\frac{1}{2\cdot 0.2}(1-\cos\psi)\right]^{1.11}=4719(2.5\cos\psi-1.5)^{1.11}$$

Eq. (7.40) $\psi_{l2}=\cos^{-1}(1-2\varepsilon_2)=\cos^{-1}(1-2\cdot 0.2)=\pm 53.13°$

ψ	$\cos\psi$	$Q_{\psi 1}$ N (lb)	$Q_{\psi 2}$ N (lb)
0°	1	21450 (4819)	4719 (1060)
±25.71°	0.9010	19980 (4488)	3442 (773)
±51.42°	0.6237	15930 (3578)	204 (46)
±77.13°	0.2227	10250 (2299)	0
±102.84°	0.2227	4321 (964)	0
±128.55°	0.6237	0	0
±154.26°	0.9010	0	0
180°	-1	0	0