

Example 8.1 Radial Deflection of a Cylindrical Roller Bearing Subjected to Applied Radial Load

Problem Statement

For the 209 CRB of Ex. 7.4 estimate the bearing radial deflection. Compare this value with δ_{\max} obtained in Ex. 7.3 assuming a diametral clearance of 0.0406 mm.

Problem Solution

Ex. (7.4) $Q_{\max} = 1589 \text{ N (357.1 lb)}$

Ex. (2.7) $l = 9.6 \text{ mm (0.3789 in.)}$

Eq. (8.4)

$$\delta_r = 7.68 \cdot 10^{-5} \frac{Q_{\max}^{0.9}}{l^{0.8} \cos \alpha} = 7.68 \cdot 10^{-5} \frac{(1589)^{0.9}}{(9.6)^{0.8} \cos 0^\circ} = 0.00953 \text{ mm}$$

$$\delta_{\max} = \delta_r + \frac{P_d}{2} = 0.00953 + \frac{0.0406}{2} = 0.02983 \text{ mm}$$

Ex. 7.3 $\delta_{\max} = 0.03251 \text{ mm} > 0.02983$

Example 8.2 Axial Deflection of an Angular-Contact Ball Bearing Subjected to Applied Axial Load

Problem Statement

For the 218 ACBB of Ex. 7.5 estimate the bearing axial deflection at 17,800 N (4000 lb) thrust load. Compare this value with δ_{\max} of Ex. 7.5

Problem Solution

Ex. (7.5) $Z = 16$ balls

Ex. (2.3) $\alpha^0 = 40^\circ$
 $D = 22.23$ mm (0.875 in.)

Eq. (7.26)
$$Q = \frac{F_a}{Z \sin \alpha} = \frac{17800}{16 \cdot \sin 40^\circ} = 1731N(388.9lb)$$

Eq. (8.5)

$$\delta_a = 4.36 \cdot 10^{-4} \frac{Q_{\max}^{2/3}}{D^{1/3} \sin \alpha} = 4.36 \cdot 10^{-4} \frac{(1731)^{2/3}}{(22.23)^{1/3} \sin 40^\circ} = 0.0348mm(0.00137in.)$$

Ex. (7.5) $\delta_a = 0.0386$ mm (This is slightly greater owing to accounting for the change in contact angle and hence maximum ball load with thrust load.)

Example 8.3 Axial Deflection of an Angular-Contact Ball Bearing Subjected to Applied Axial Load

Problem Statement

A duplex pair of back-to-back mounted 218 ACBBs as shown in Fig. 8.3 is axially preloaded to 4450 N (1000 lb). Determine the axial deflection under 8900 N (2000 lb) applied thrust load.

Problem Solution

Ex. (2.3) $\alpha^0 = 40^\circ$
 $D = 22.23 \text{ mm (0.875 in.)}$
 $B = 0.0464$

Ex. (7.5) $Z = 16 \text{ balls}$
 $K = 896.7 \text{ N/mm}^2 \text{ (130,000 psi)}$

Eq. (8.16) $\frac{F_p}{ZD^2K} = \sin \alpha_p \left(\frac{\cos \alpha^0}{\cos \alpha_p} - 1 \right)^{1.5}$

(a) $\frac{4450}{16 \cdot (22.23)^2 \cdot 896.7} = \sin \alpha_p \left(\frac{\cos 40^\circ}{\cos \alpha_p} - 1 \right)^{1.5}$

Solving (a) using the Newton-Raphson method yields $\alpha_p = 40.66^\circ$

Eq. (8.17) $\delta_p = \frac{BD \sin(\alpha_p - \alpha^0)}{\cos \alpha_p}$

$$\delta_p = \frac{0.0464 \cdot 22.23 \sin(40.66^\circ - 40^\circ)}{\cos 40.66^\circ} = 0.01555 \text{ mm (0.0006122 in.)}$$

Eq. (8.13) $\frac{F_a}{ZD^2K} = \sin \alpha_1 \left(\frac{\cos \alpha^0}{\cos \alpha_1} - 1 \right)^{1.5} - \sin \alpha_2 \left(\frac{\cos \alpha^0}{\cos \alpha_2} - 1 \right)^{1.5}$

$$\frac{8900}{16 \cdot (22.23)^2 \cdot 896.7} = \sin \alpha_1 \left(\frac{\cos 40^\circ}{\cos \alpha_1} - 1 \right)^{1.5} - \sin \alpha_2 \left(\frac{\cos 40^\circ}{\cos \alpha_2} - 1 \right)^{1.5}$$

$$(b) \quad 0.001255 = \sin \alpha_1 \left(\frac{0.7660}{\cos \alpha_1} - 1 \right)^{1.5} - \sin \alpha_2 \left(\frac{0.7660}{\cos \alpha_2} - 1 \right)^{1.5}$$

$$\text{Eq. (8.15)} \quad \frac{\sin(\alpha_1 - \alpha^0)}{\cos \alpha_1} + \frac{\sin(\alpha_2 - \alpha^0)}{\cos \alpha_2} = \frac{2\delta_p}{BD}$$

$$(c) \quad \frac{\sin(\alpha_1 - 40^\circ)}{\cos \alpha_1} + \frac{\sin(\alpha_2 - 40^\circ)}{\cos \alpha_2} = \frac{2 \cdot 0.01555}{0.0464 \cdot 22.23} = 0.03014$$

Solving (b) and (c) simultaneously using the Newton-Raphson method gives $\alpha_1 = 41.09^\circ$ and $\alpha_2 = 40.22^\circ$.

$$\delta_a = \delta_{a1} - \delta_p = \frac{BD \sin(\alpha_1 - \alpha^0)}{\cos \alpha_1}$$

$$\delta_a = \frac{0.0464 \cdot 22.23 \sin(41.09^\circ - 40^\circ)}{\cos 41.09^\circ} = 0.01039 \text{ mm} (0.0004091 \text{ in.})$$

This value may be compared against the axial deflection for a single 218 angular-contact bearing subjected to 8900 N (2000 lb) thrust load. Under this condition $\delta_a = 0.02446$ mm (0.000963 in.). Thus, the preloaded bearing set is more than twice as stiff as the single bearing under the applied thrust load. Such stiffness improvement is utilized in machine tool bearing applications.

Example 8.4 Radial Deflection of a Cylindrical Roller Bearing Subjected to Applied Radial Load

Problem Statement

The 209 CRB of Ex. 7.3 is manufactured with a tapered bore and driven up a tapered shaft as in Fig. 8.9(b) until a radial interference of 0.00254 mm (0.0001 in.) occurs. For a radial load of 4450 N (1000 lb), determine:

- * maximum roller load
- * extent of the load zone
- * radial deflection

Compare these results with those of Ex. 7.3

Problem Solution

$$\text{Eq. (7.22)} \quad F_r = ZK_n \left(\delta_r - \frac{1}{2} P_d \right)^{10/9} J_r(\varepsilon)$$

$$\frac{F_r}{ZK_n} = \frac{4450}{14 \cdot 2.72 \cdot 10^5} = 0.001169$$

$$0.001169 = \left(\delta_r - \frac{(-0.00254)}{2} \right)^{10/9} J_r(\varepsilon)$$

(a)

$$(\delta_r + 0.00127)^{10/9} J_r(\varepsilon) = 0.001169$$

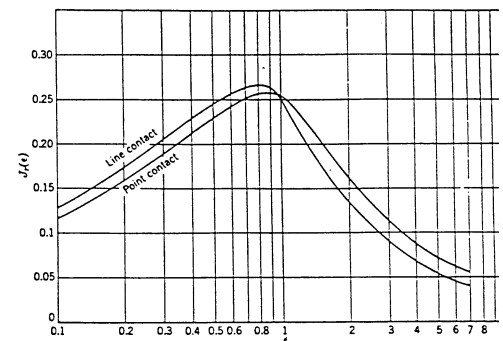
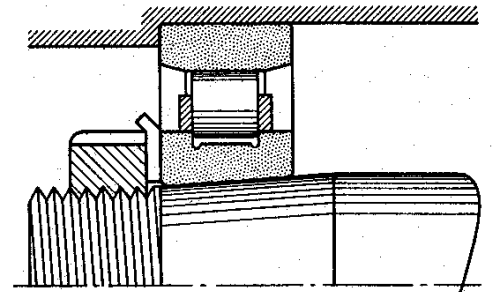
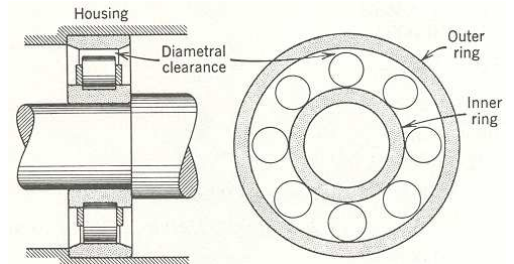
$$\text{Eq. (7.12)} \quad \varepsilon = \frac{1}{2} \left(1 - \frac{P_d}{2\delta_r} \right)^{1.11}$$

$$\text{(b)} \quad \varepsilon = \frac{1}{2} \left(1 - \frac{(-0.00254)}{2\delta_r} \right)^{1.11} = 0.5 - \frac{0.000635}{\delta_r}$$

Use Fig. 7.2 and Eq. (a) & (b) to solve for δ_r .

$$\delta_r = 0.00660 \text{ mm} \quad \varepsilon = 0.596 \quad J_r(0.596) = 0.256$$

CRB with clearance



$$\text{Eq. (7.19)} \quad F_r = ZQ_{\max} J_r(\varepsilon)$$

$$Q_{\max} = \frac{F_r}{ZJ_r(\varepsilon)} = \frac{4450}{14 \cdot 0.256} = 1242\text{N}(279\text{lb})$$

$$\text{Eq. (7.13)}$$

$$\psi_l = \cos^{-1}\left(\frac{P_d}{2\delta_r}\right) = \cos^{-1}\left(\frac{0.00254}{2 \cdot 0.0066}\right) = \pm 101.53^\circ$$

COMPARISONS

Parameter	Ex. 7.3	Ex. 8.4
P_d (mm)	0.0406	-0.0025
Q_{\max} (N)	1915	1242
δ_r (mm)	0.032	0.0066
ψ_l ($^\circ$)	± 50.58	± 101.53

Example 8.5 Limiting Thrust Load of an Angular-Contact Ball Bearing

Problem Statement

The outer ring land diameter of the 218 ACBB of Ex. 7.5 is 133.8 mm. Estimate the bearing thrust load that will cause the balls to override the outer ring land.

Problem Solution

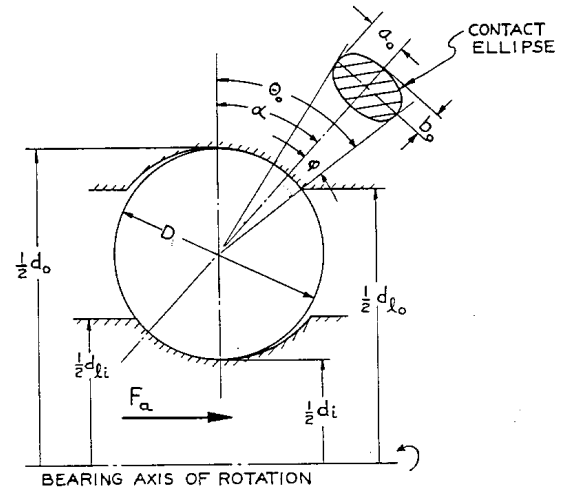
Ex. (2.3) $\alpha^0 = 40E$
 $D = 22.23 \text{ mm (0.875 in.)}$
 $B = 0.0464$

$$\Sigma \rho_o = 0.09396 - \frac{0.01596 \cos \alpha}{1 + 0.1774 \cos \alpha}$$

$$d_o = 147.7 \text{ mm (5.816 in.)}$$

Ex. (2.6) $d_m = 125.3 \text{ mm (4.932 in.)}$

Ex. (7.5) $Z = 16 \text{ balls}$
 $K = 896.7 \text{ N/mm}^2 \text{ (130,000 psi)}$



Eq. (8.21) $\theta_o = \cos^{-1} \left(1 - \frac{d_o - d_{lo}}{D} \right) = \cos^{-1} \left(1 - \frac{147.7 - 133.8}{22.23} \right) = 67.68^\circ$

Eq. (2.27) $\gamma = \frac{D \cos \alpha}{d_m} = \frac{22.23 \cos \alpha}{125.3} = 0.1774 \cos \alpha$

Eq. (2.30) $\Sigma \rho_o = \frac{1}{D} \left(4 - \frac{1}{f_o} - \frac{2\gamma}{1 + \gamma} \right) = \frac{1}{22.23} \left(4 - \frac{1}{0.5232} - \frac{2 \cdot 0.1774 \cos \alpha}{1 + 0.1774 \cos \alpha} \right)$

$$\Sigma \rho_o = 0.09396 - \frac{0.01596 \cos \alpha}{1 + 0.1774 \cos \alpha}$$

$$\text{Eq. (2.31)} \quad F(\rho)_o = \frac{\frac{1}{f_o} - \frac{2\gamma}{1+\gamma}}{D\Sigma\rho_o} = \frac{\frac{1}{0.5232} - \frac{2 \cdot 0.1774 \cos \alpha}{1+0.1774 \cos \alpha}}{22.23 \cdot \left(0.09396 - \frac{0.01596 \cos \alpha}{1+0.1774 \cos \alpha} \right)}$$

$$F(\rho)_o = \frac{1,911 - \frac{0.3548 \cos \alpha}{1+0.1774 \cos \alpha}}{2.089 - \frac{0.3548 \cos \alpha}{1+0.1774 \cos \alpha}}$$

$$\text{Eq. (8.24)} \quad \sin(\theta - \alpha) = \frac{0.0472 a_o^* K^{1/3} \left(\frac{\cos \alpha^0}{\cos \alpha} - 1 \right)^{1/2}}{(D\Sigma\rho_o)^{1/3}}$$

$$\sin(67.98^\circ - \alpha) = \frac{0.0472 a_o^* (896.7)^{1/3} \left(\frac{\cos 40^\circ}{\cos \alpha} - 1 \right)^{1/2}}{(22.23 \Sigma \rho_o)^{1/3}}$$

$$\sin(67.98^\circ - \alpha) = \frac{0.454 a_o^* \left(\frac{0.7660}{\cos \alpha} - 1 \right)^{1/2}}{(22.23 \Sigma \rho_o)^{1/3}}$$

Solve by trial and error using Fig. 6.4. See Table below.

Assumed α	$\cos\alpha$	$\Sigma\rho$ (mm^{-1})	$F(\rho)$	a_o^*	Calculated α
45°	0.7071	0.0839	0.9046	3.11	48.58 °
47°	0.6820	0.0843	0.9050	3.12	44.13°
46°	0.6947	0.0841	0.9048	3.11	46.35°
46.5°	0.6884	0.0842	0.9049	3.12	46.32°

Use $\alpha = 46.35^\circ$

$$\text{Eq. (7.33)} \quad \frac{F_{ao}}{ZD^2K} = \sin \alpha \left(\frac{\cos \alpha^0}{\cos \alpha} - 1 \right)^{3/2}$$

$$F_{ao} = ZD^2K \sin \alpha \left(\frac{\cos \alpha^0}{\cos \alpha} - 1 \right)^{3/2} = 16 \cdot (22.23)^2 \cdot 896.7 \sin 46.35^\circ \left(\frac{\cos 40^\circ}{\cos 46.35^\circ} - 1 \right)^{3/2} = 1.87 \cdot 10^5 \text{ N}$$